

INTRODUCTION TO INELASTIC NEUTRON SCATTERING

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1. Diffraction



2. Can we learn more ?



3. Cross sections



4. In practice TOF



5. In pratice TAS



6. Polarization



7. ESS and polychromatic pulses



8. More ...



7. Tonight





<https://www.neutron-sciences.org/>

Articles by

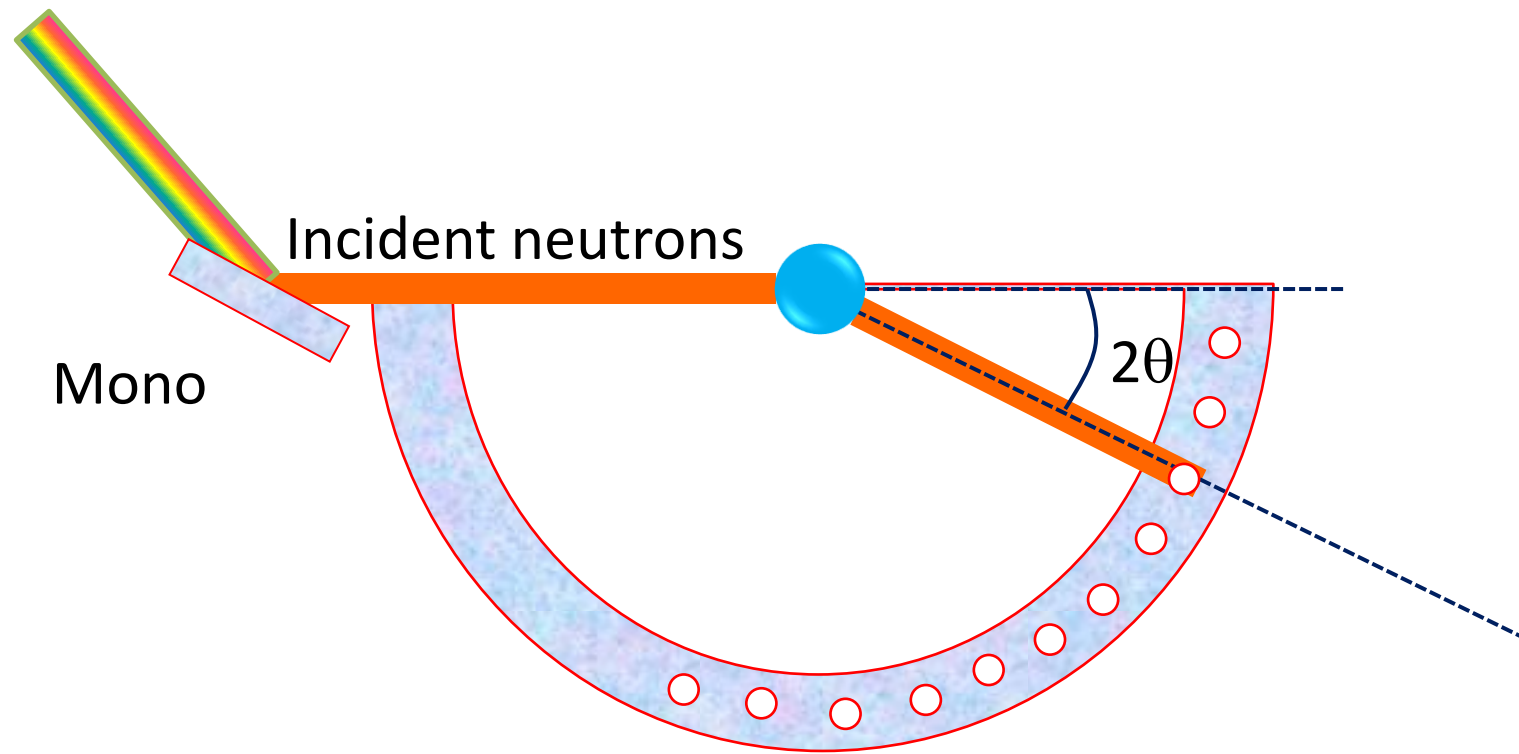
Eric Ressouche, Mechthilde Enderle, Stéphane Raymond

And others ...

1. Diffraction



Diffraction

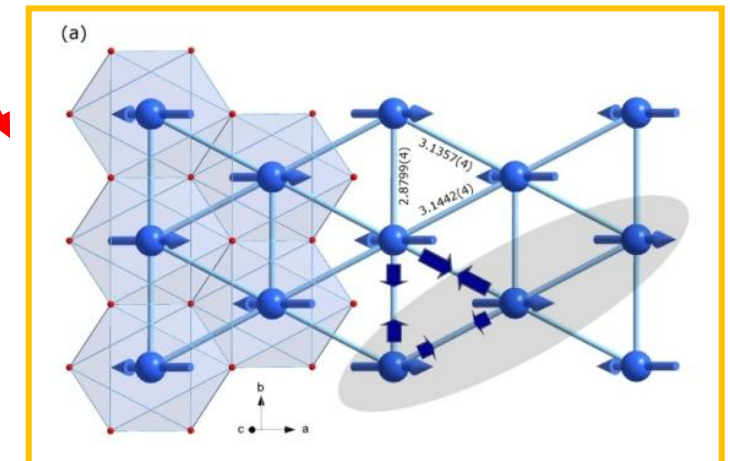
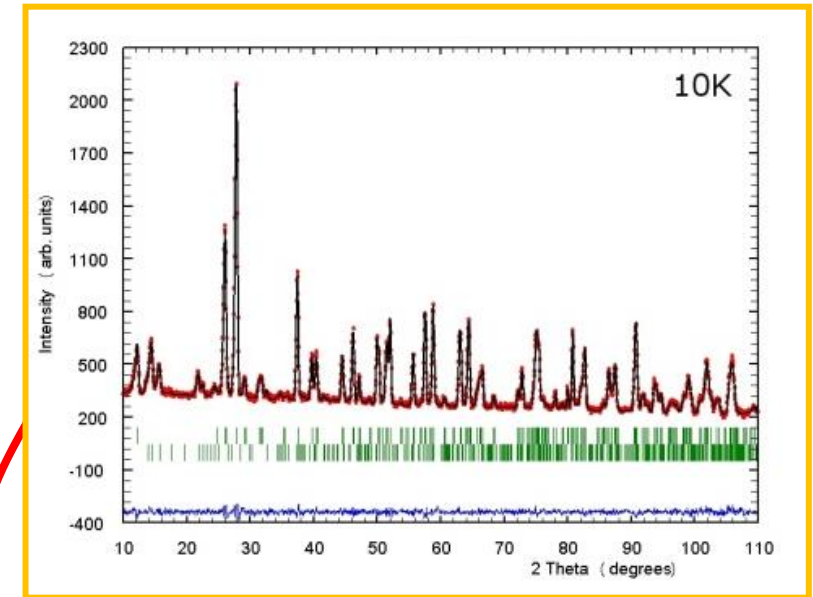


Mono

Incident neutrons

2θ

« simple » and robust



FUNDAMENTAL FORMULAS 1 & 2

$$F_N(\mathbf{Q}) = \sum_{\ell} b_{\ell} e^{i\mathbf{Q}\mathbf{r}_{\ell}} e^{-W_{\ell}}$$

Strong interaction with nuclei

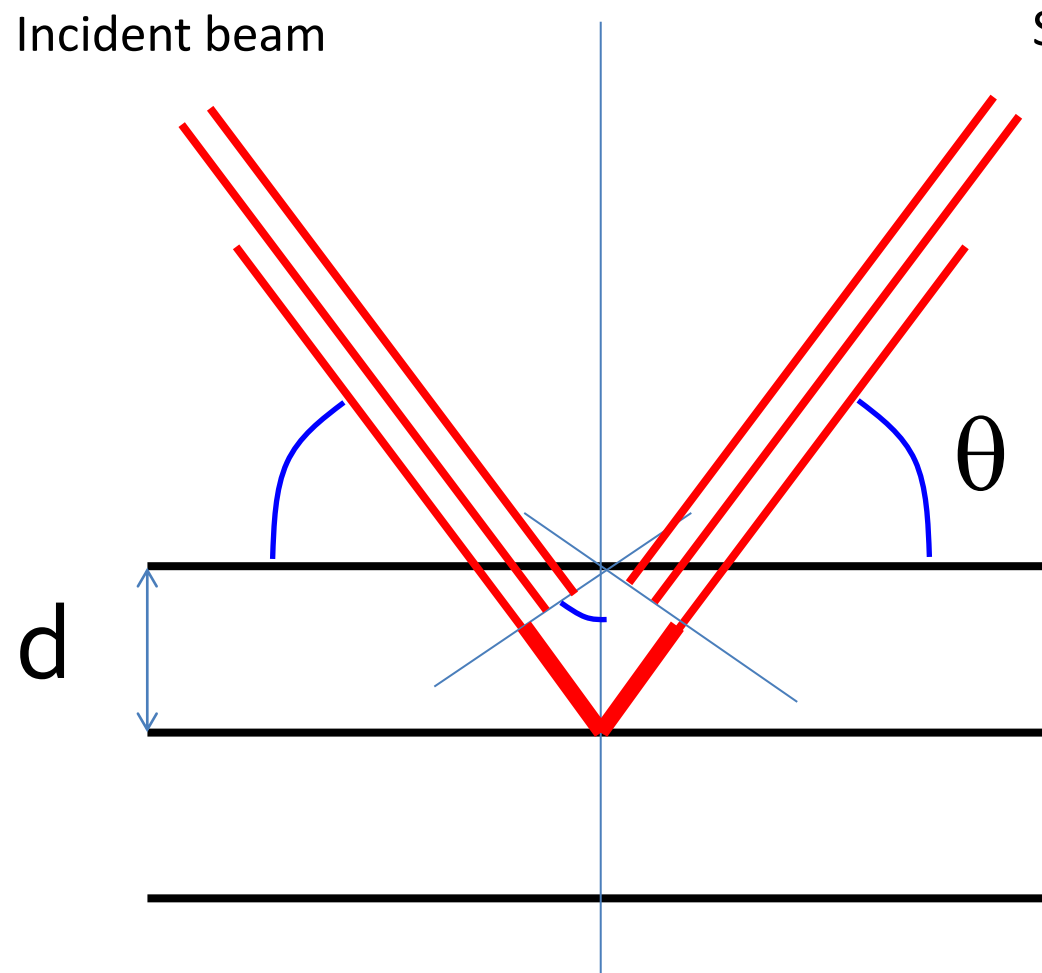
$$\mathbf{F}_M(\mathbf{Q}) = \sum_{\ell} \mathbf{S}_{\ell\perp} e^{i\mathbf{Q}\mathbf{r}_{\ell}} e^{-W_{\ell}}$$

The spin of the neutron interacts with the dipolar field created by the spin (and orbital motion) of unpaired electrons

2. Can we learn more ?



Can we learn more ?

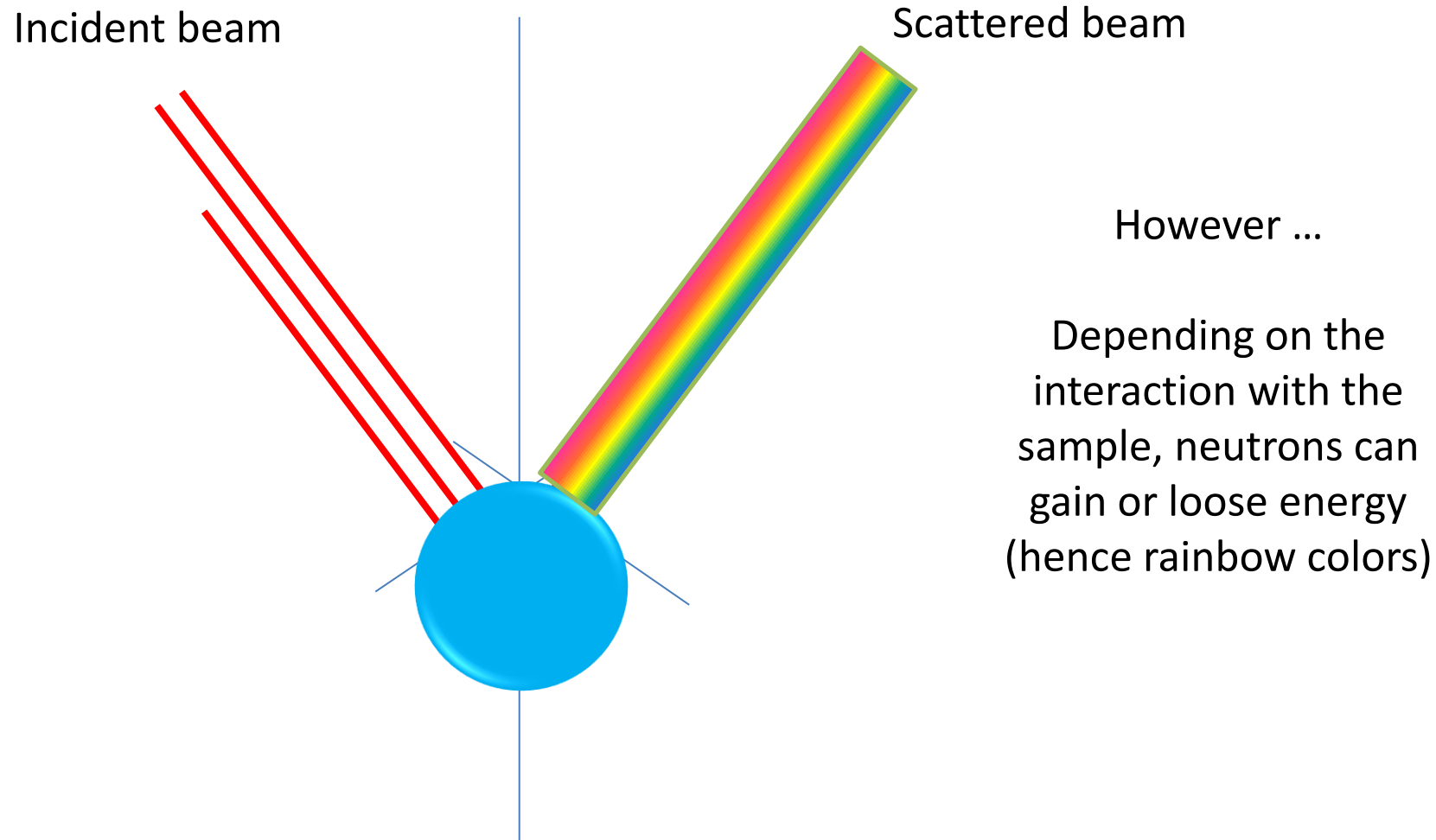


Bragg law

$$\lambda = 2d \sin \theta$$

Diffraction is an
elastic process...

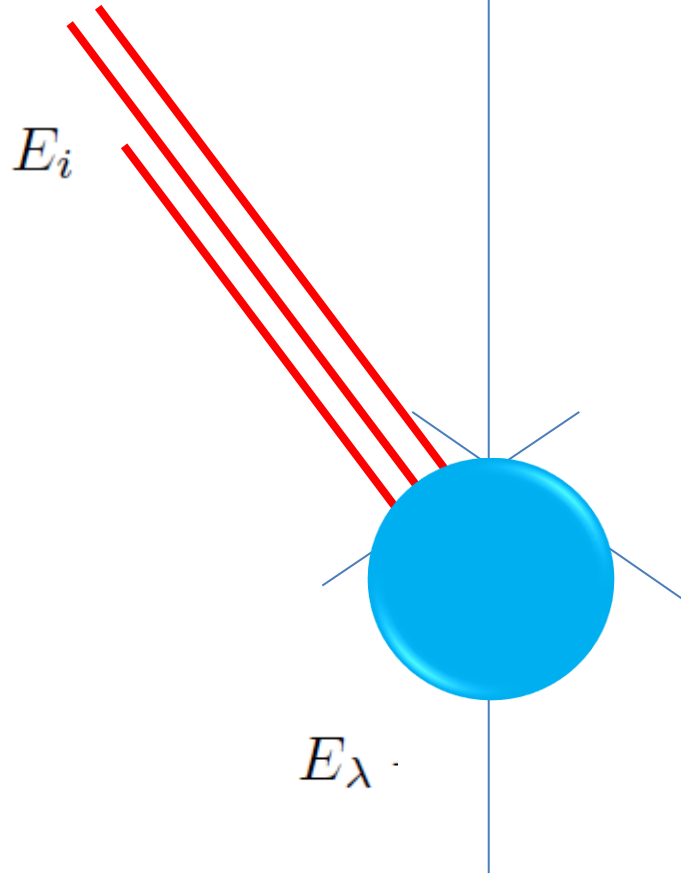
Can we learn more ?



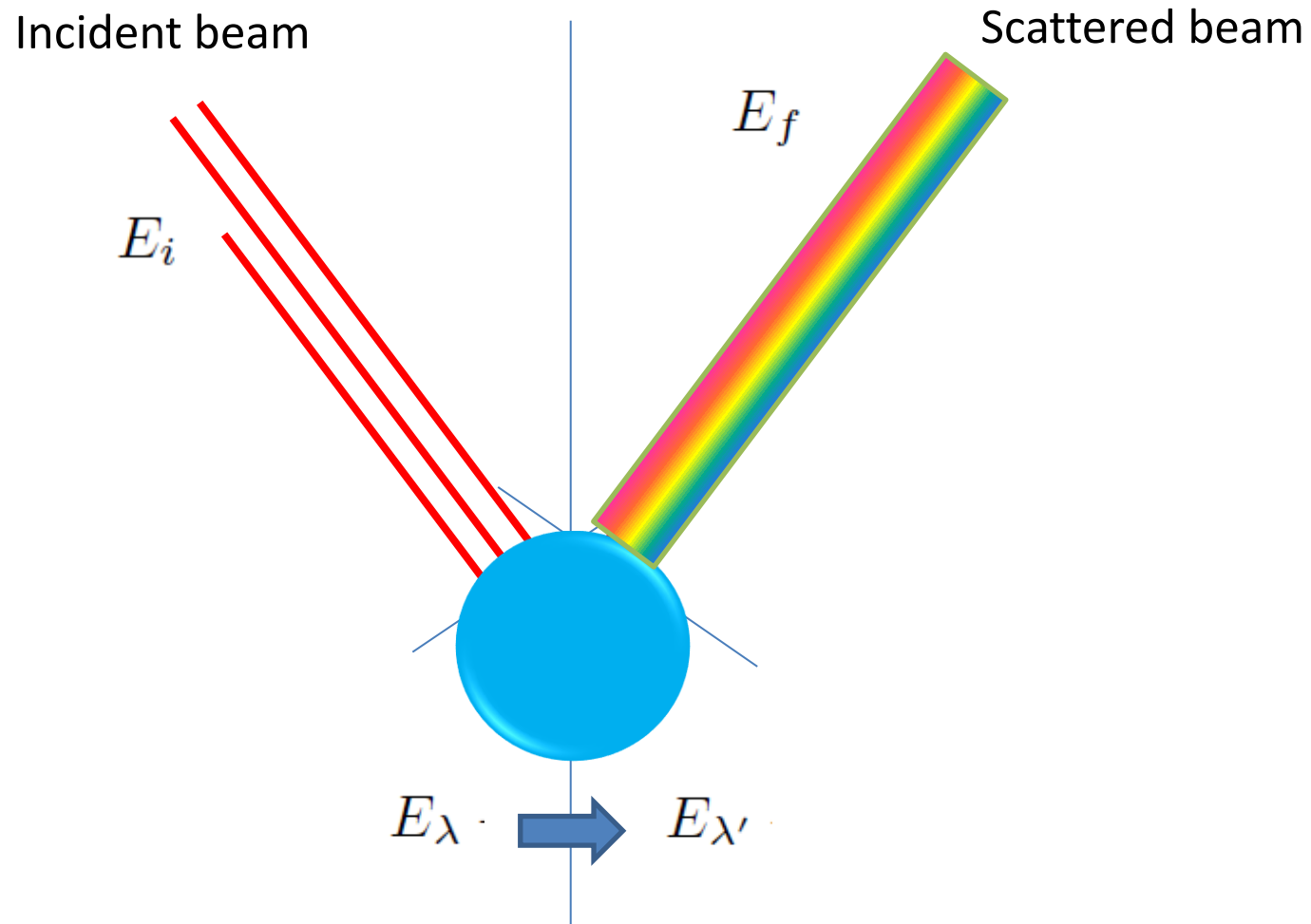
Can we learn more ?

Incident beam

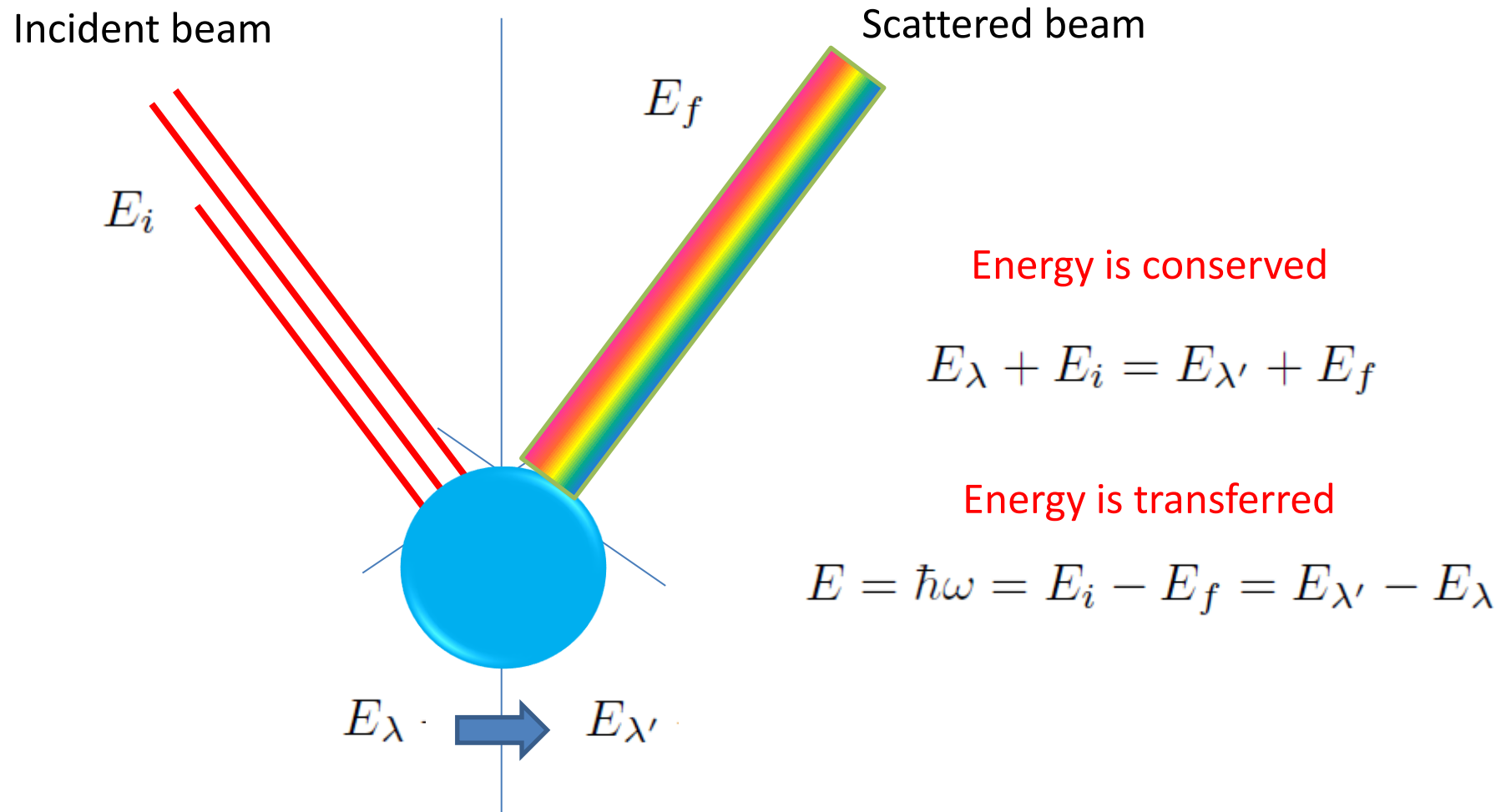
Scattered beam



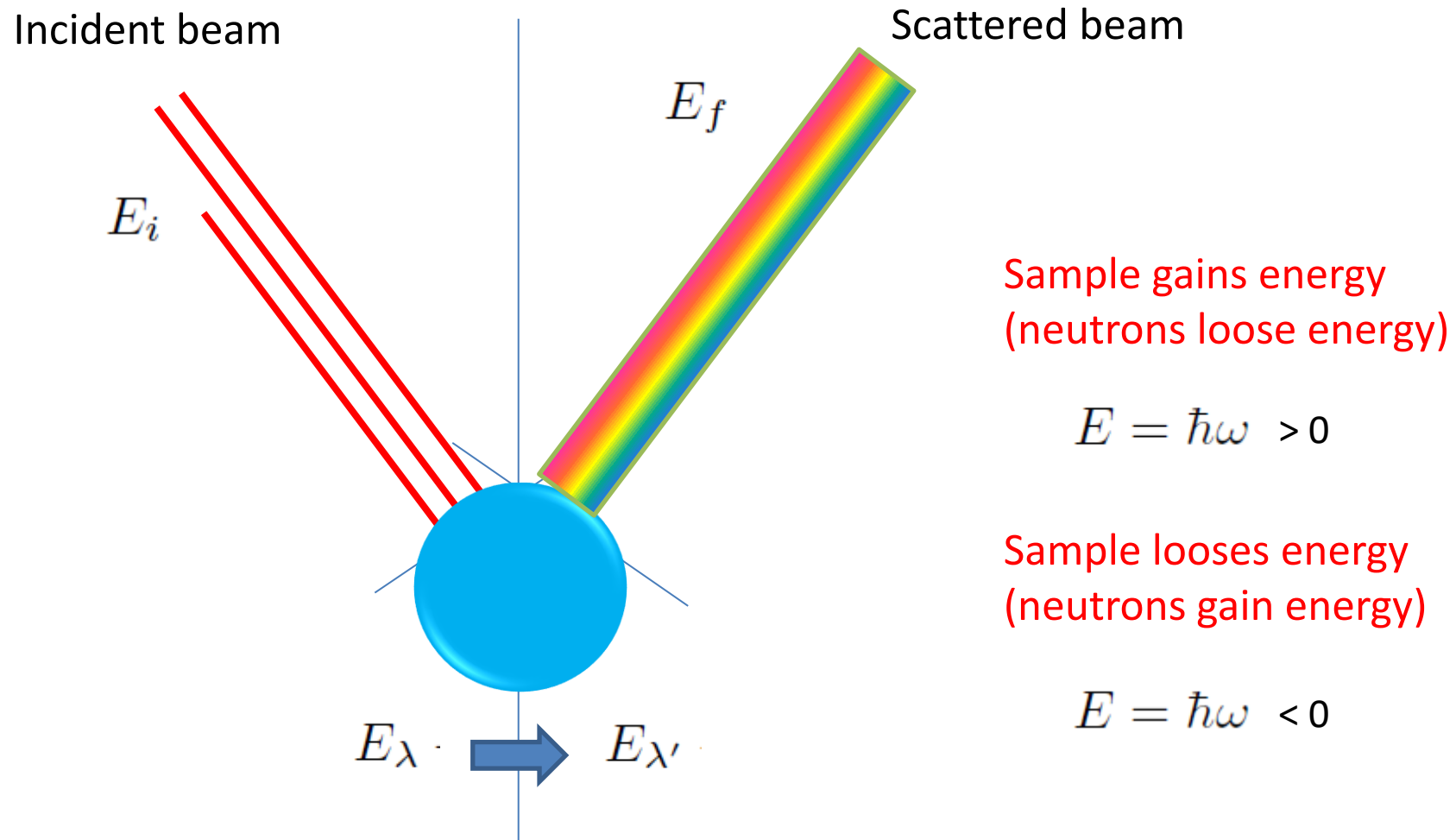
Can we learn more ?



Can we learn more ?

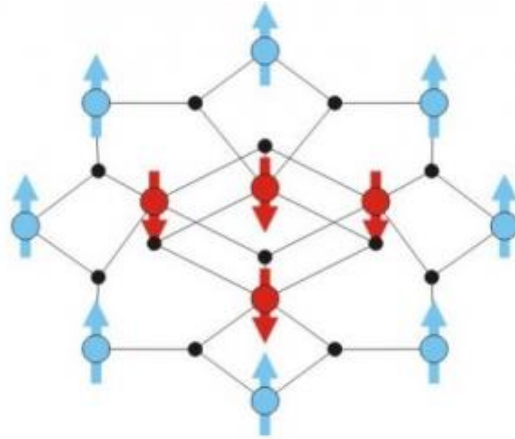


Can we learn more ?

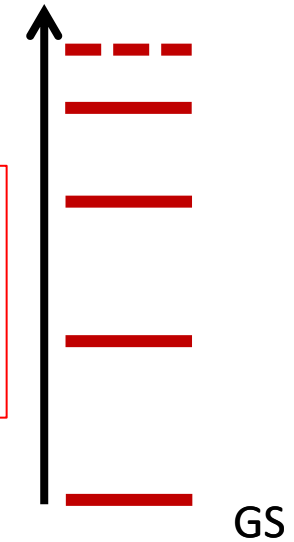


Example 1 : Mn12-acetate

Mn12-acetate : a molecular nanomagnet



Each molecule is a quantum object characterized by a discrete spectrum

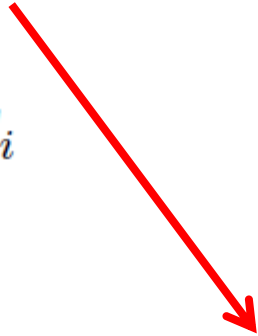


- The ground state is actually a ferrimagnetic arrangement
- Possible magnetization reversal by thermal effects, but also by quantum effects

Example 1 : Mn12-acetate

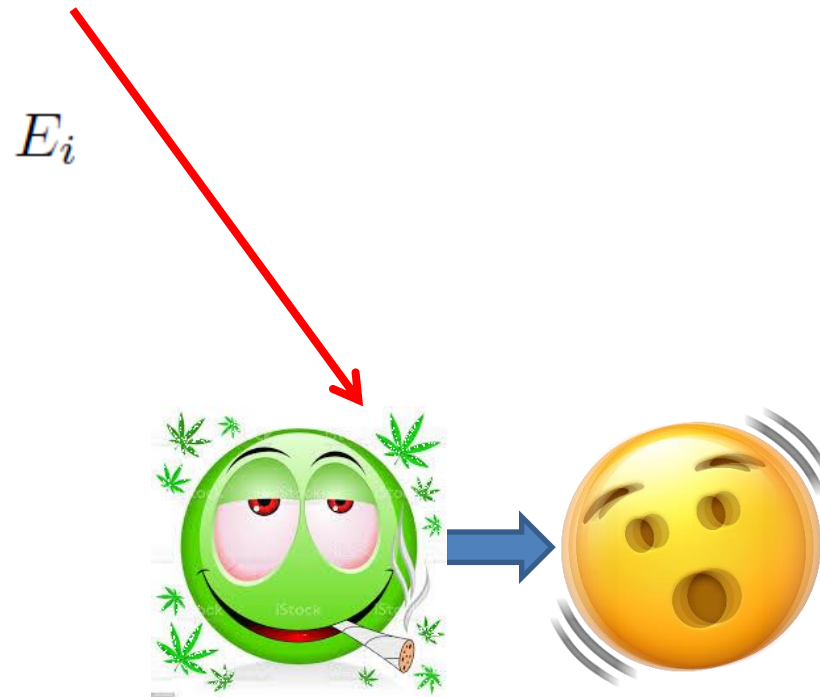
T= 0 (in the ground state) :

E_i



Example 1 : Mn12-acetate

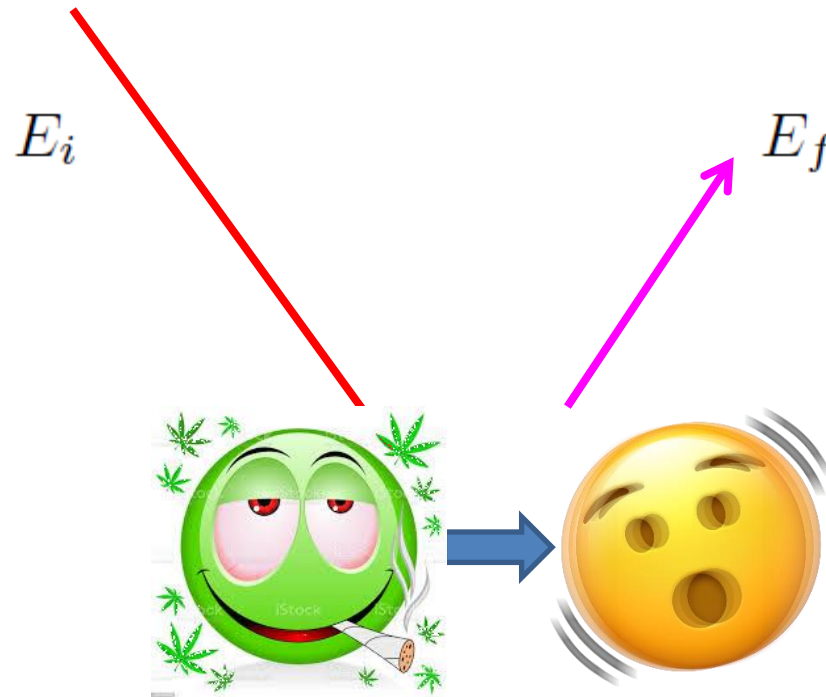
T= 0 (in the ground state) :



- Gets excited by populating an excited state with energy E

Example 1 : Mn12-acetate

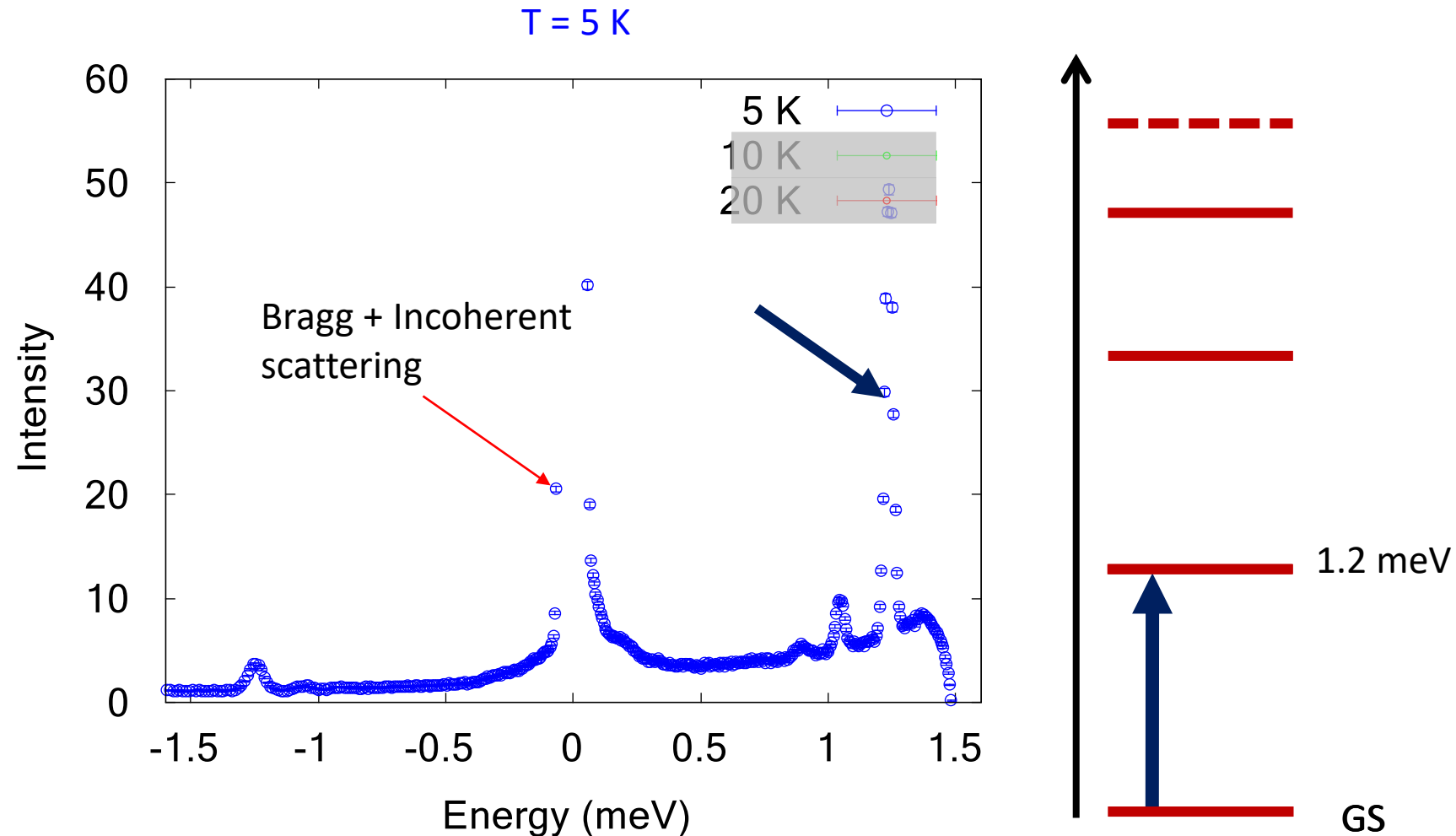
T= 0 (in the ground state) :



- Gets excited by populating an excited state with energy E
- The outgoing neutron has less energy

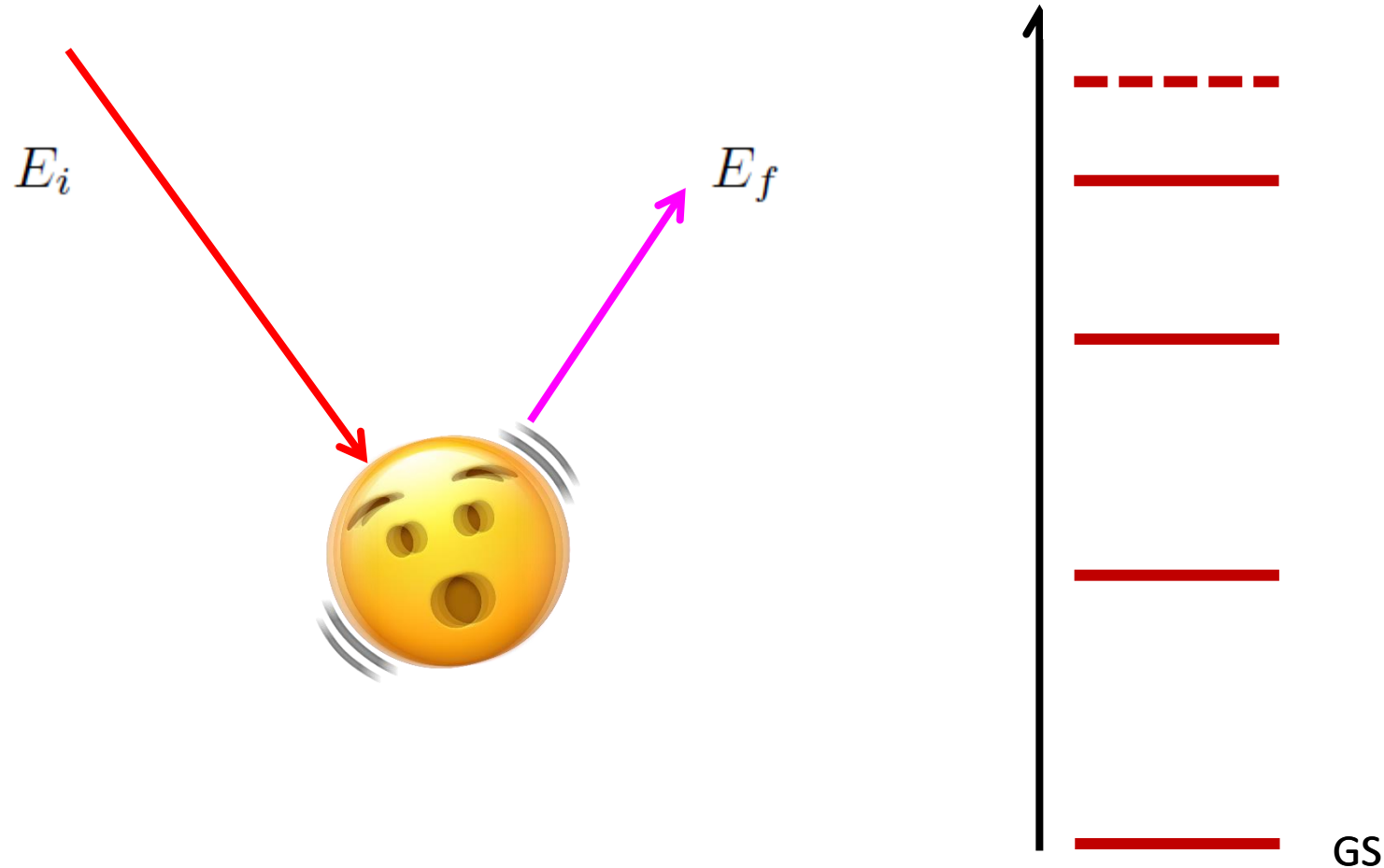
$$E = \hbar\omega = E_i - E_f$$

Example 1 : Mn12-acetate



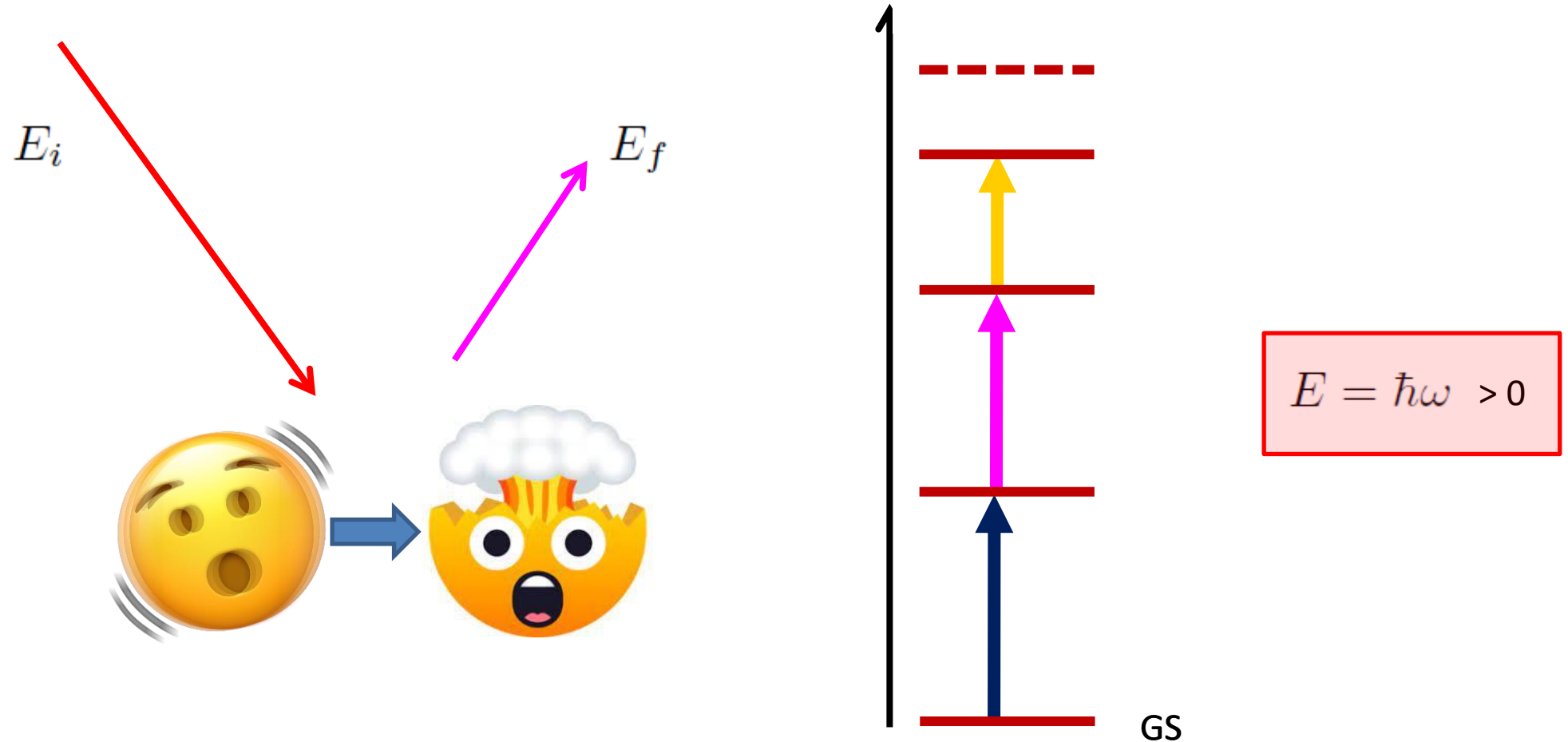
Example 1 : Mn12-acetate

At finite T, excited states are populated up to Boltzmann factor $\frac{\exp -E_n/k_B T}{\sum_n \exp -E_n/k_B T}$

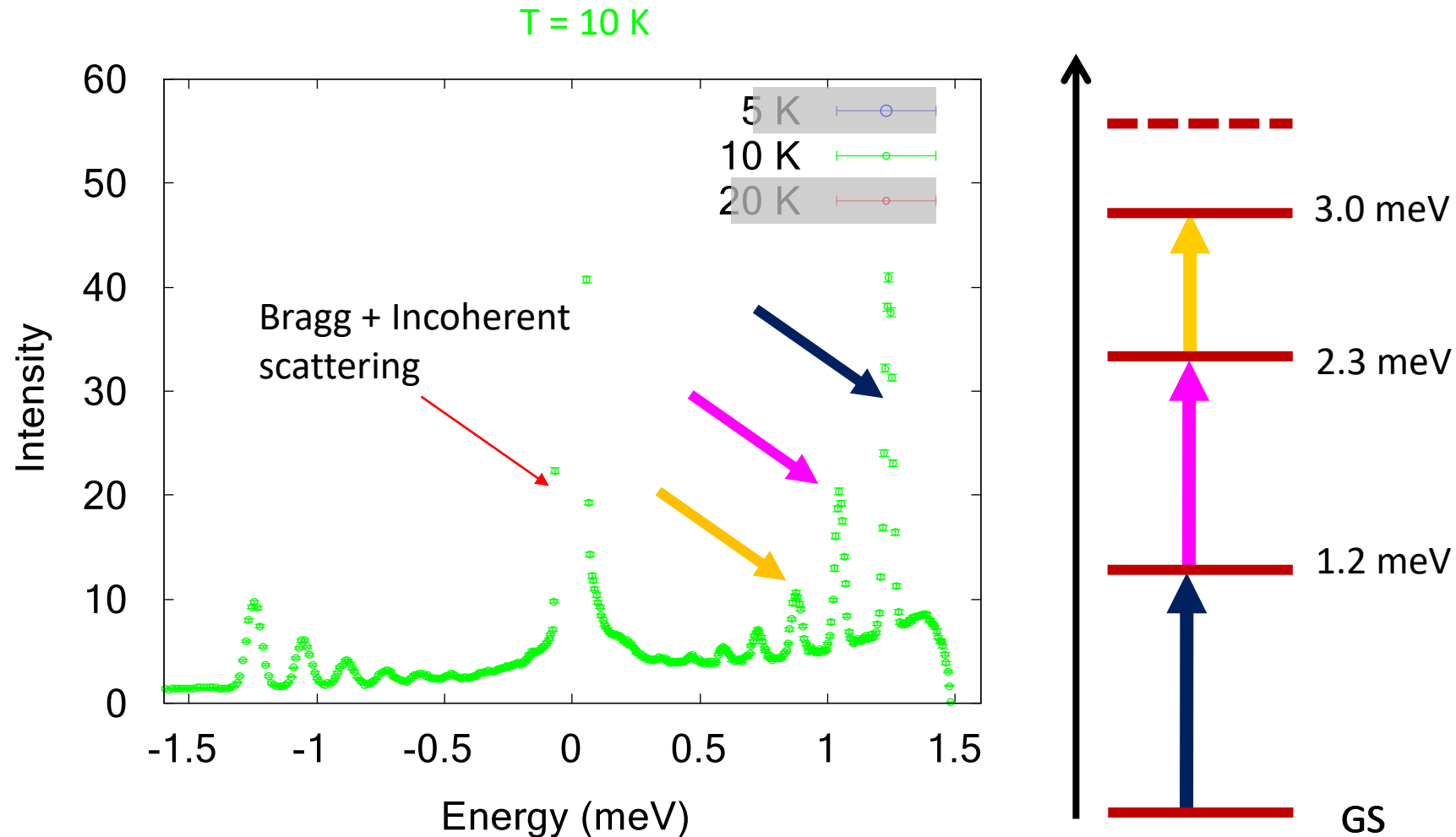


Example 1 : Mn12-acetate

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Example 1 : Mn12-acetate

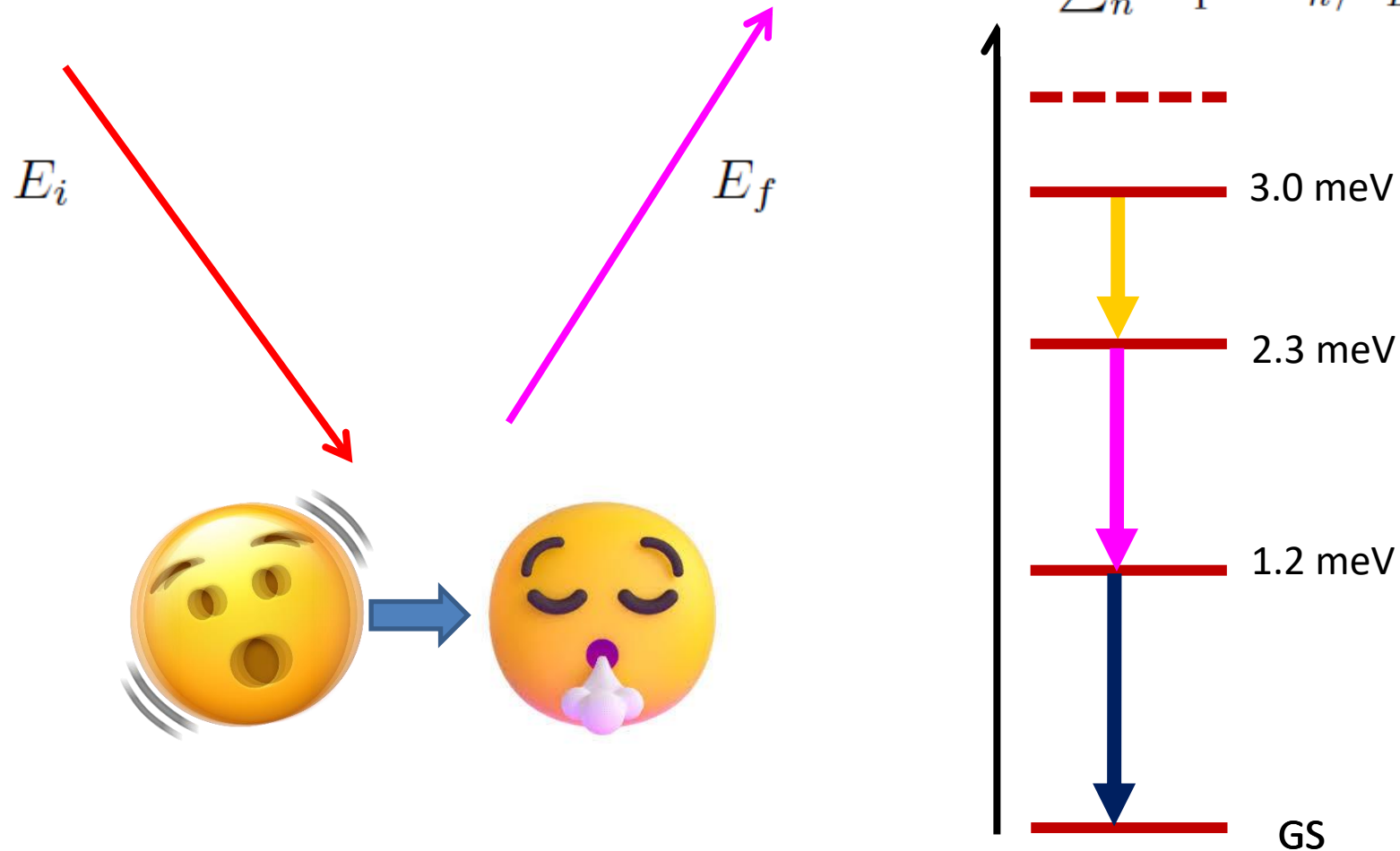


$$E = \hbar\omega > 0$$

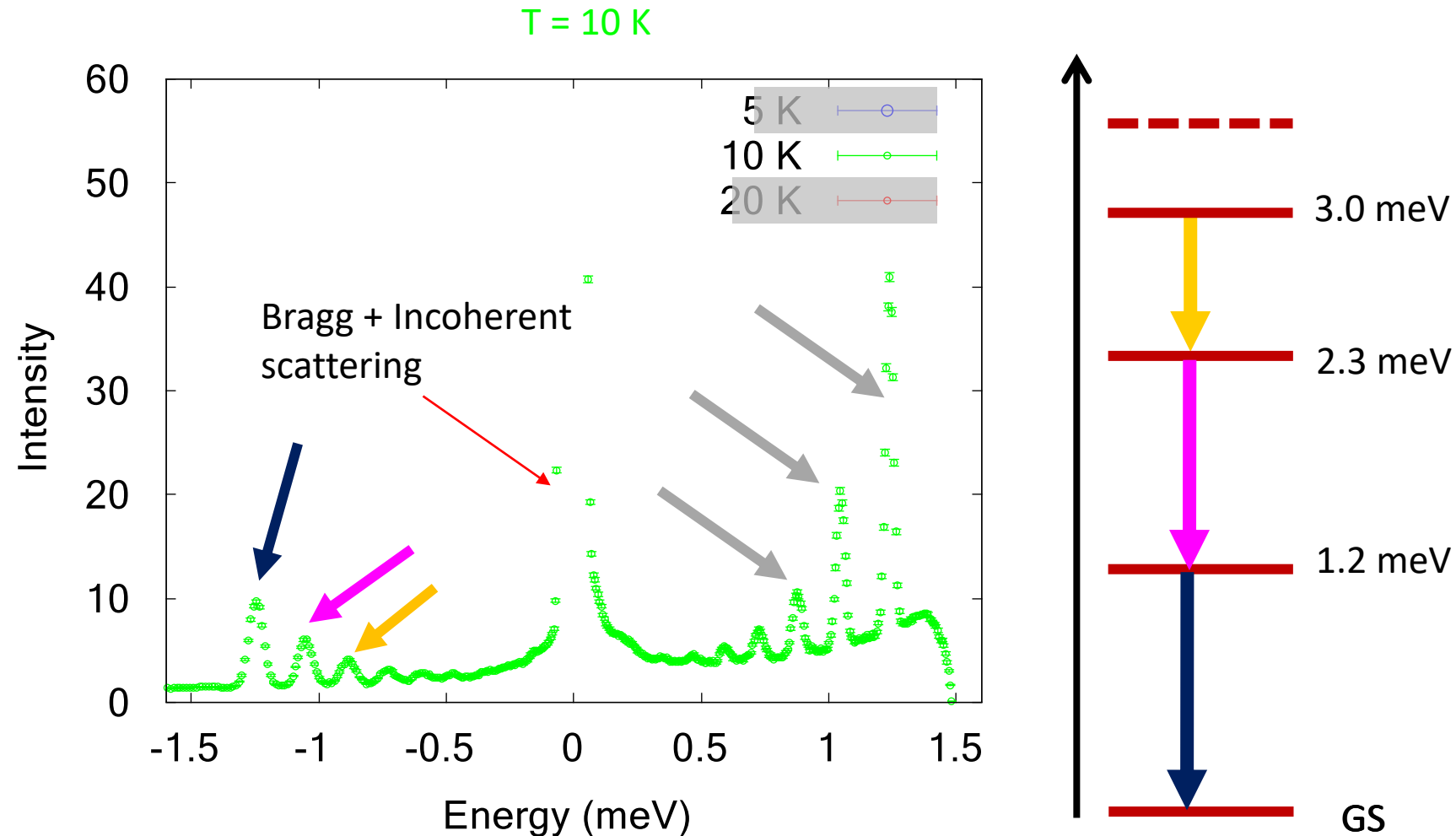
Example 1 : Mn12-acetate

At finite T, excited states are populated up to Boltzmann factor

$$\frac{\exp -E_n/k_B T}{\sum_n \exp -E_n/k_B T}$$

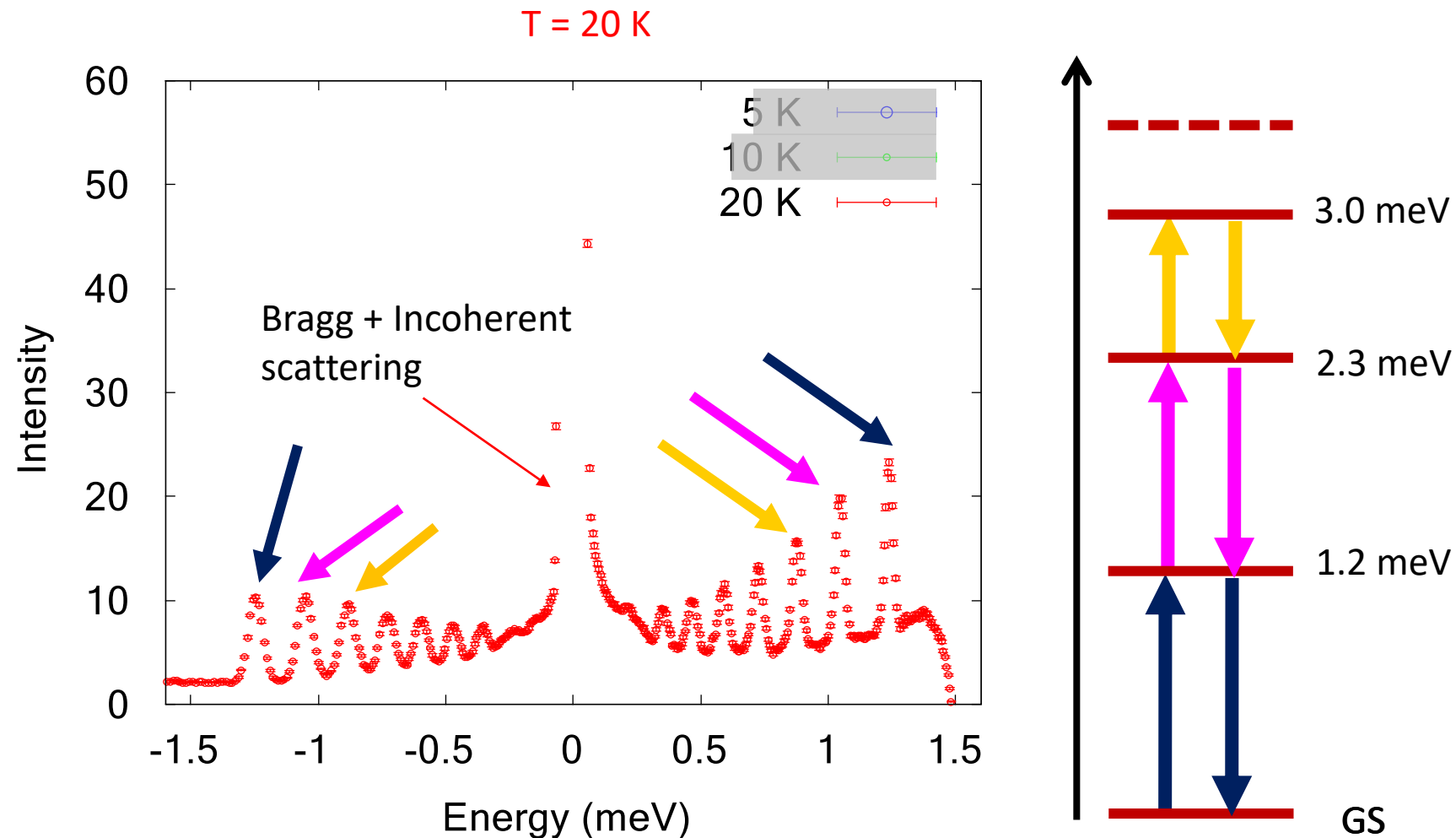


Example 1 : Mn12-acetate



$$E = \hbar\omega < 0$$

Example 1 : Mn12-acetate

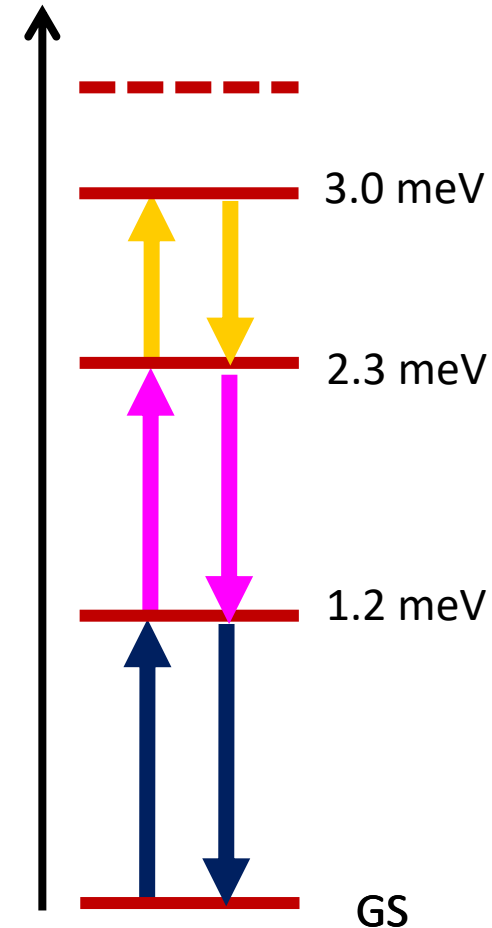


Example 1 : Mn12-acetate

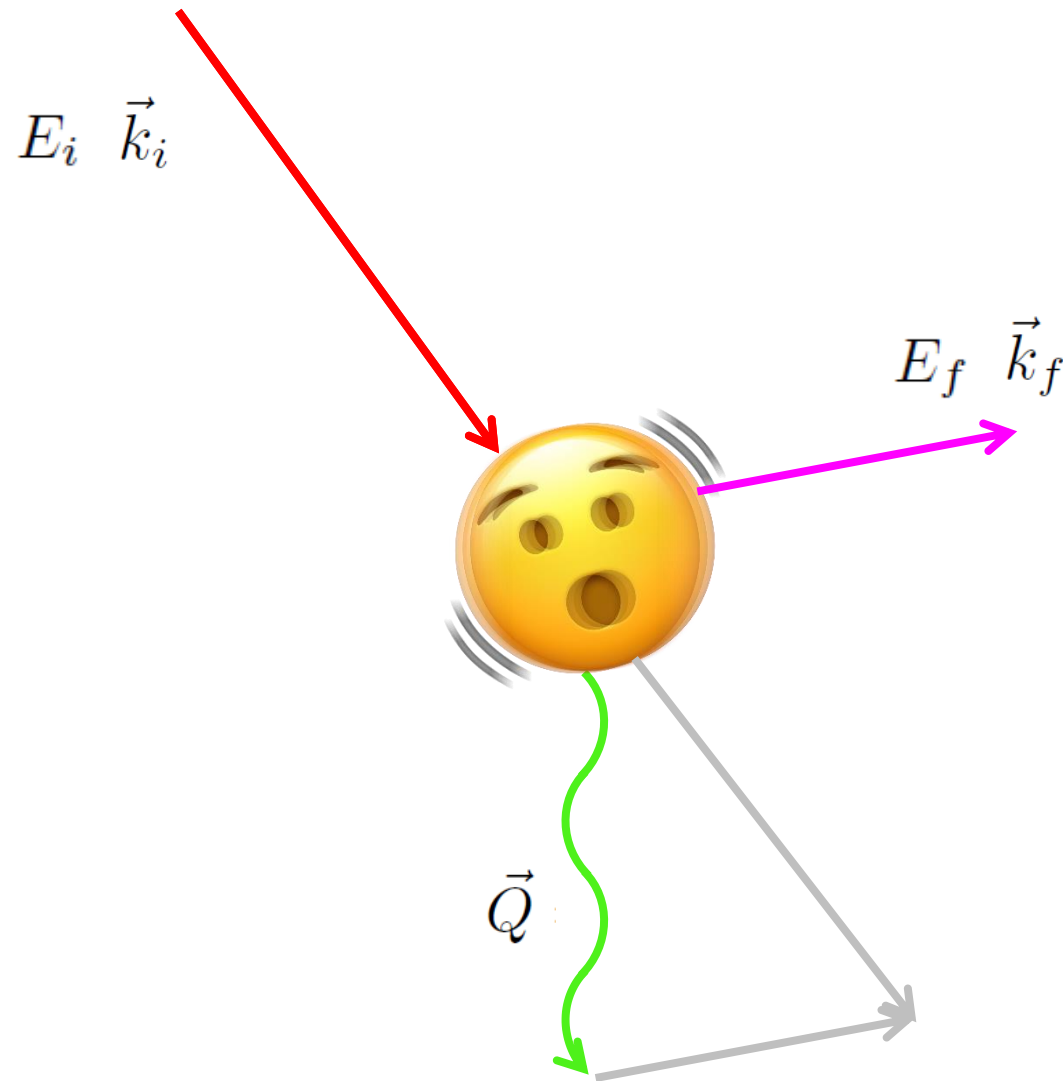
Neutrons induce transitions from a given quantum state to another:

- The energy of such a transition is the energy difference between two quantum levels
- With increasing T, more and more levels get populated, hence more and more transitions become visible.

$$\frac{\exp -E_n/k_B T}{\sum_n \exp -E_n/k_B T}$$



Example 2 : Phonons

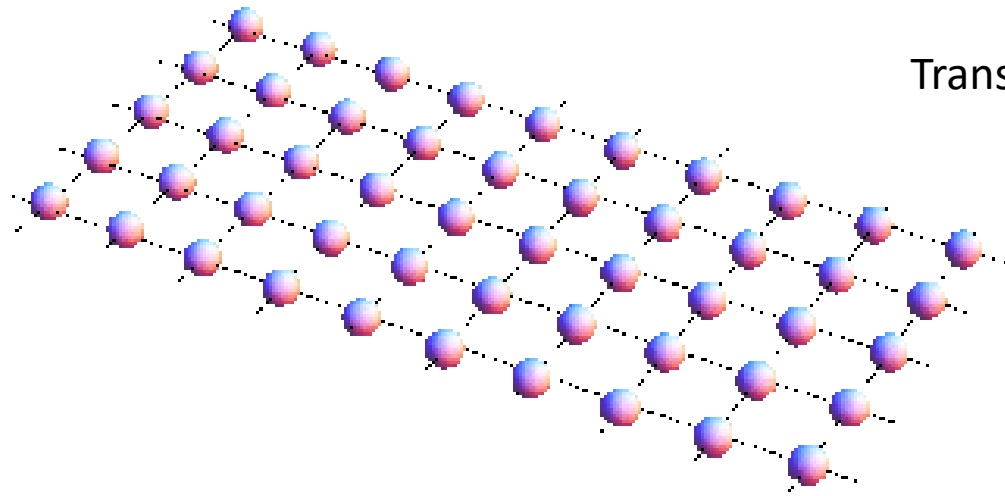


- Gets excited by creating a phonon with energy E and momentum Q

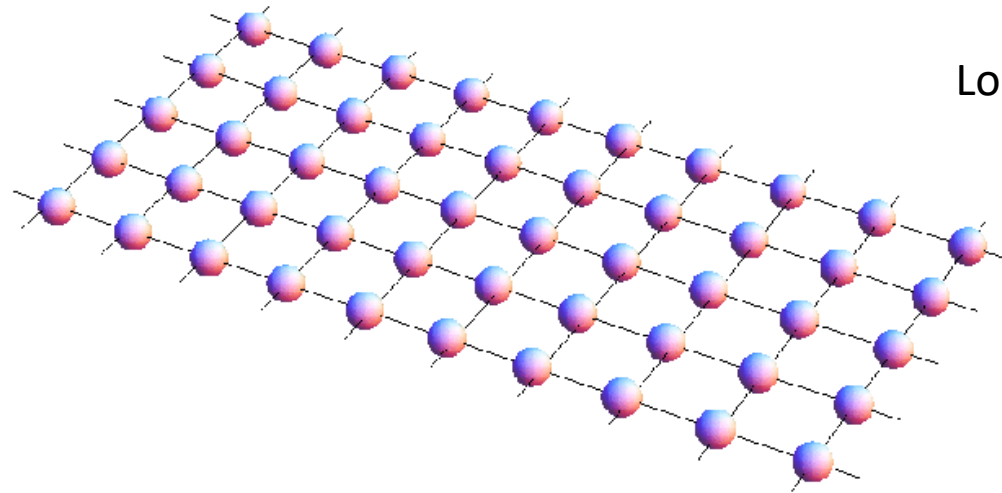
$$E = \hbar\omega = E_i - E_f$$

$$\vec{Q} = \vec{k}_i - \vec{k}_f$$

Example 2 : Phonons



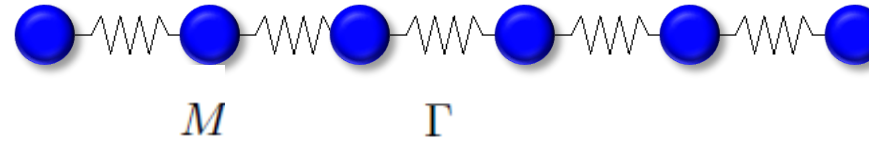
Transverse



Longitudinal

Example 2 : Phonons

Monoatomic chain



$$M \frac{d^2}{dt^2} u_m = \Gamma (u_{m+1} + u_{m-1} - 2u_m)$$

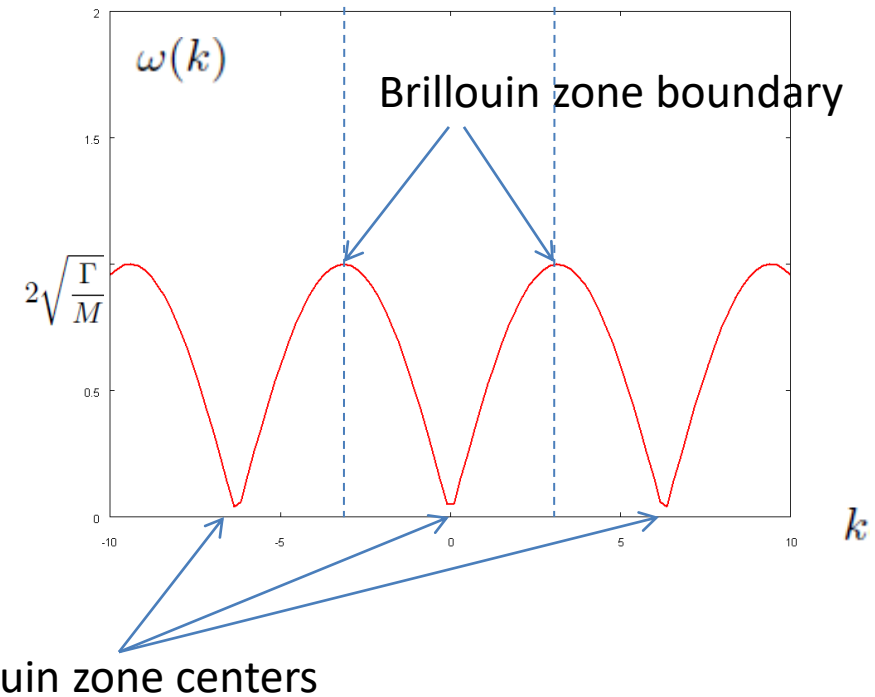
Force constant between atoms

Fourier Transform

$$-M\omega^2 = \Gamma (e^{ika} + e^{-ika} - 2)$$

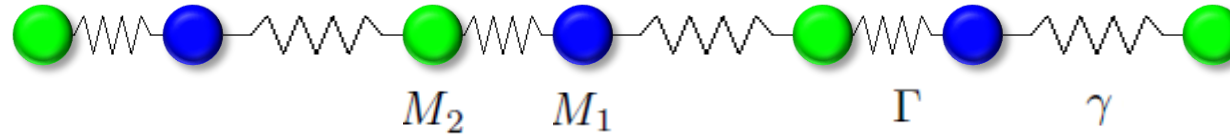
$$\omega(k) = 2\sqrt{\frac{\Gamma}{M}} \sin \frac{ka}{2}$$

Dispersion relation



Example 2 : Phonons

Di-atomic chain



$$M_1 \frac{d^2}{dt^2} u_{m,1} = \Gamma (u_{m,2} - u_{m,1}) + \gamma (u_{m-1,2} - u_{m,1})$$

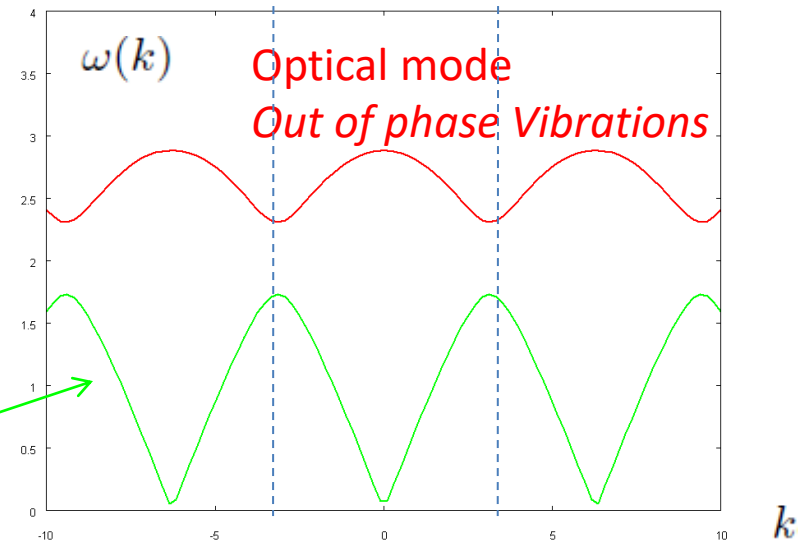
$$M_2 \frac{d^2}{dt^2} u_{m,2} = \Gamma (u_{m,1} - u_{m,2}) + \gamma (u_{m+1,1} - u_{m,2})$$

Force constants
between atoms

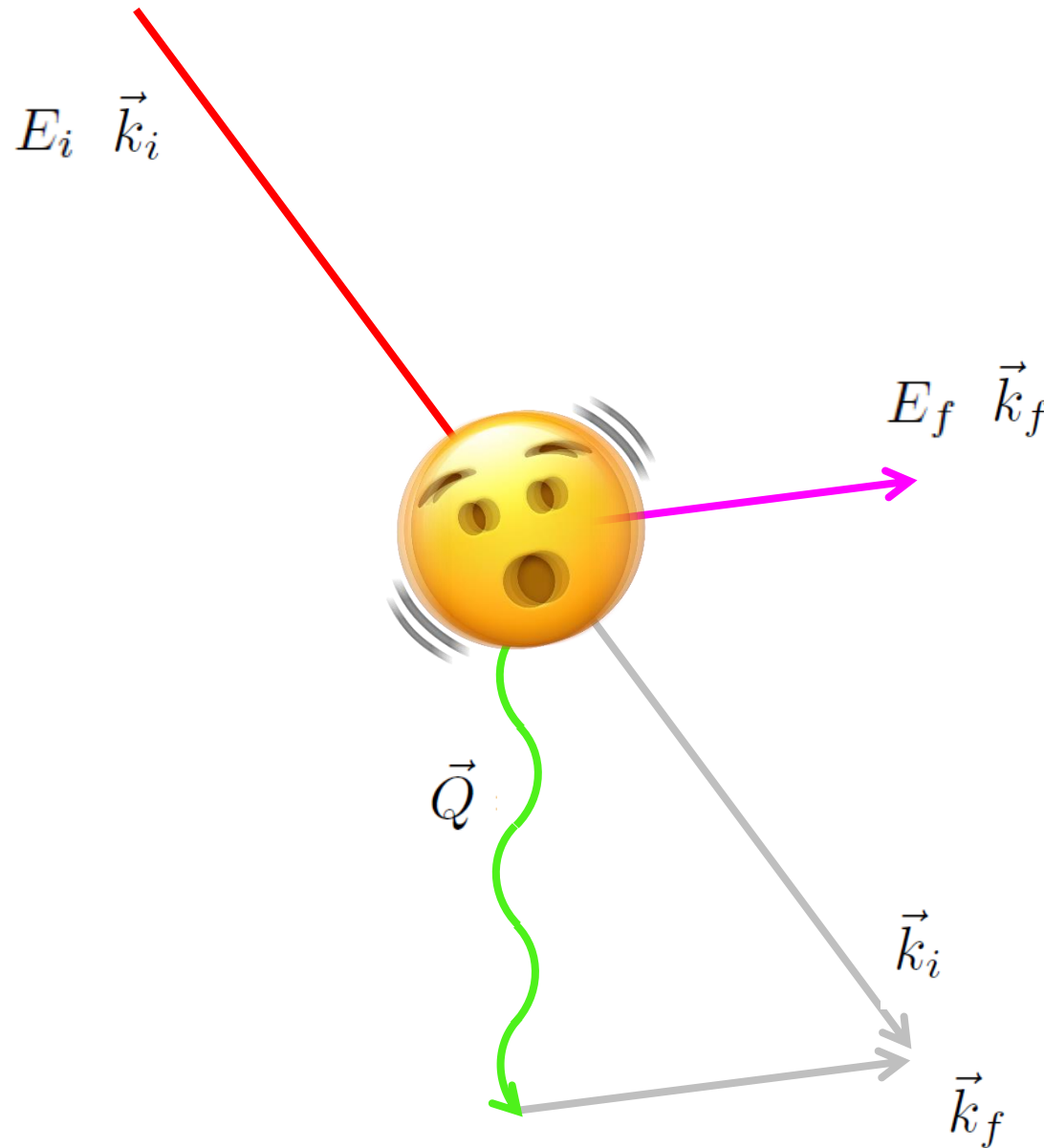
Fourier transform: for each k , eigenvalue problem

$$\begin{pmatrix} \omega^2 - \frac{\Gamma + \gamma}{M_1} & \frac{\Gamma + \gamma e^{-ika}}{\sqrt{M_1 M_2}} \\ \frac{\Gamma + \gamma e^{ika}}{\sqrt{M_1 M_2}} & \omega^2 - \frac{\Gamma + \gamma}{M_2} \end{pmatrix} \begin{pmatrix} \sqrt{M_1} u_1 \\ \sqrt{M_2} u_2 \end{pmatrix} = 0$$

Acoustic mode
In phase Vibrations

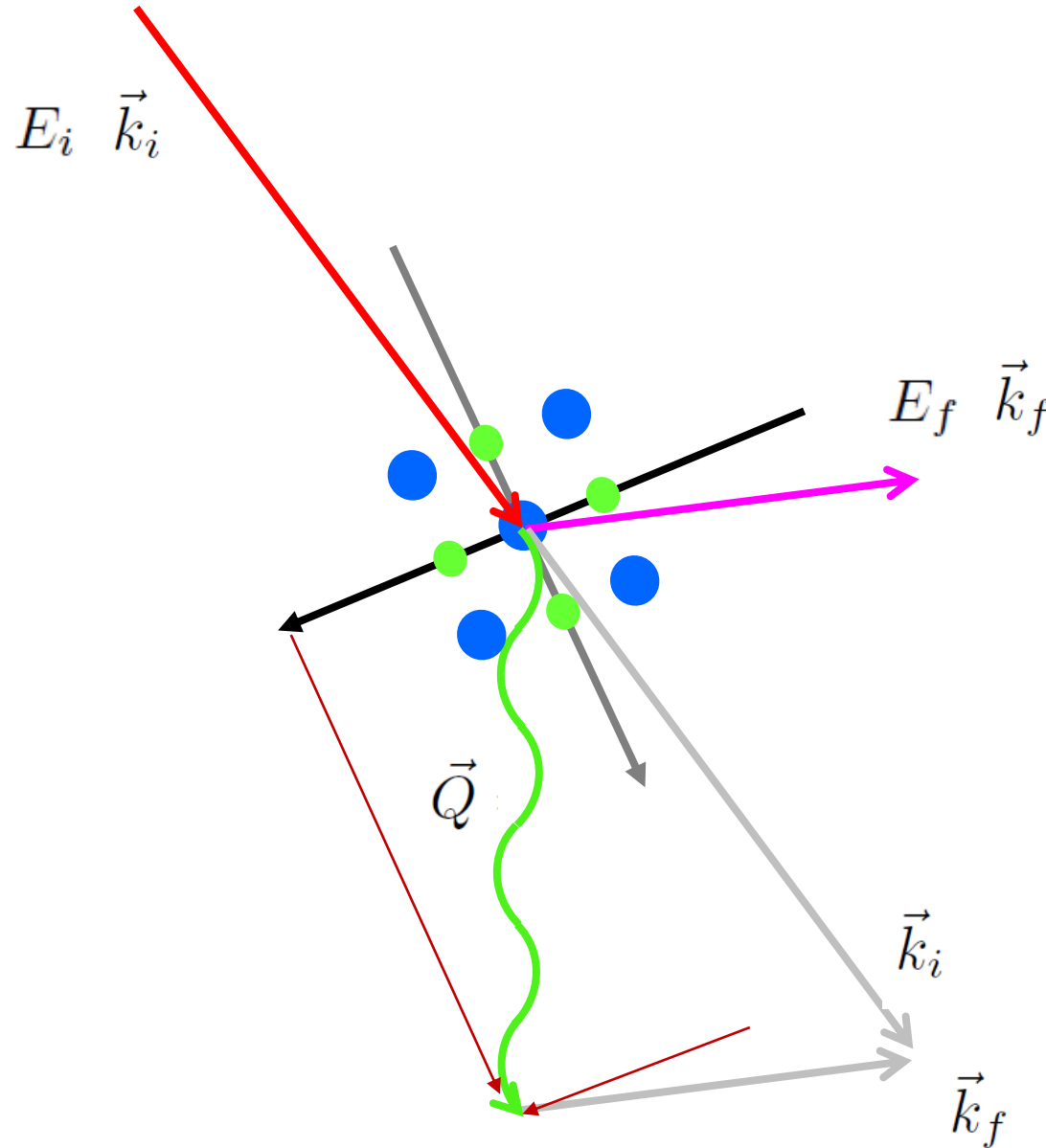


Single crystal

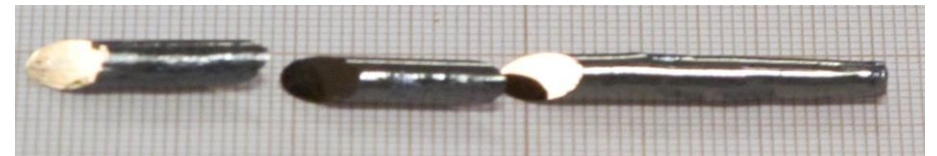


- Gets excited by creating an excitation with energy E and momentum Q
- Q is a vector
- single crystal sample

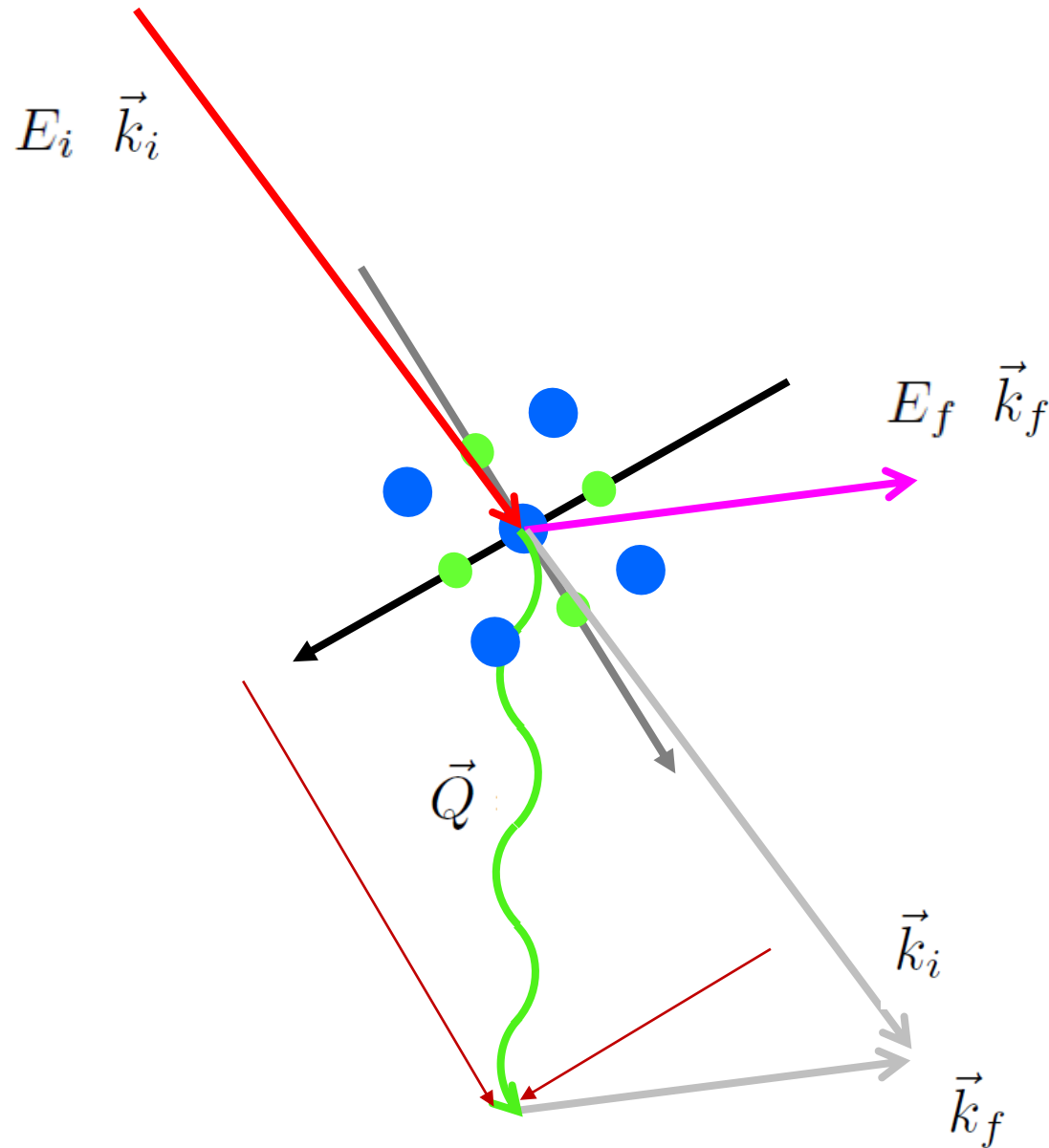
Single crystal



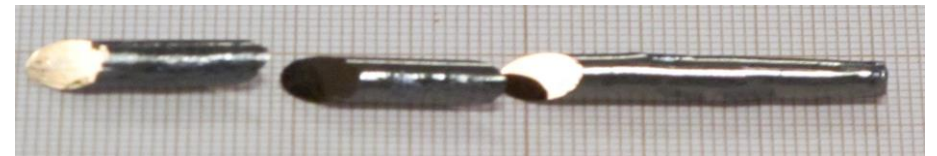
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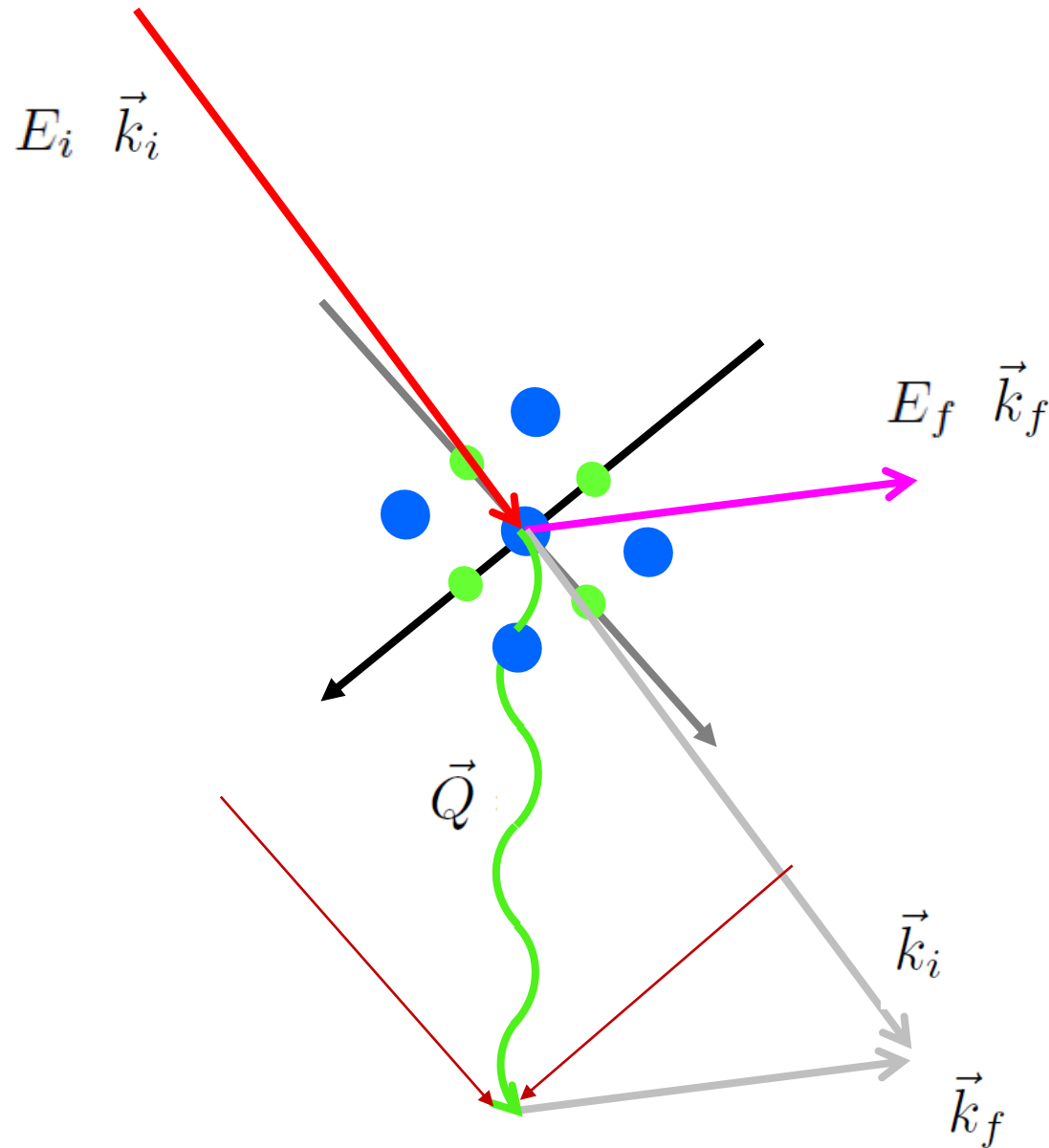
Single crystal



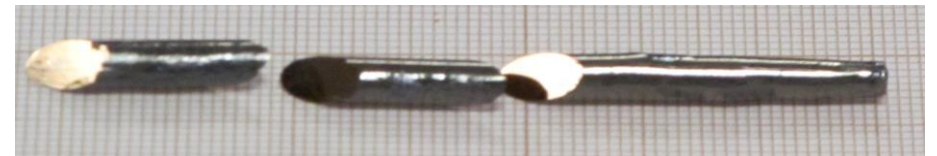
- Rotate the sample to select the components of $\vec{Q}$ with respect to the sample's lattice



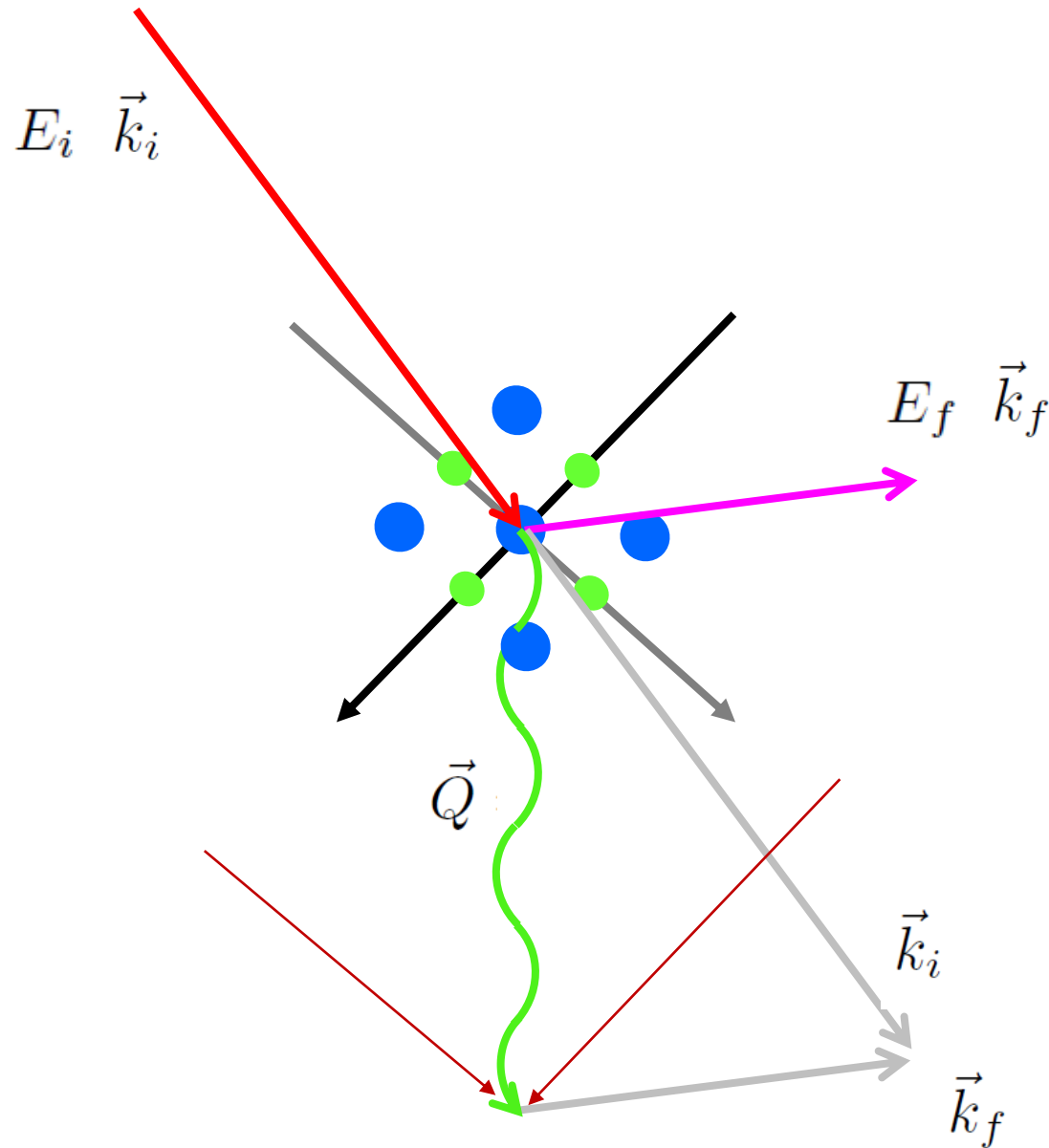
Single crystal



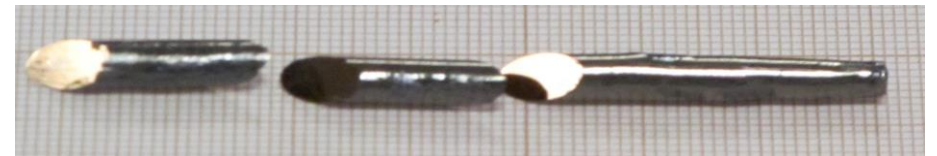
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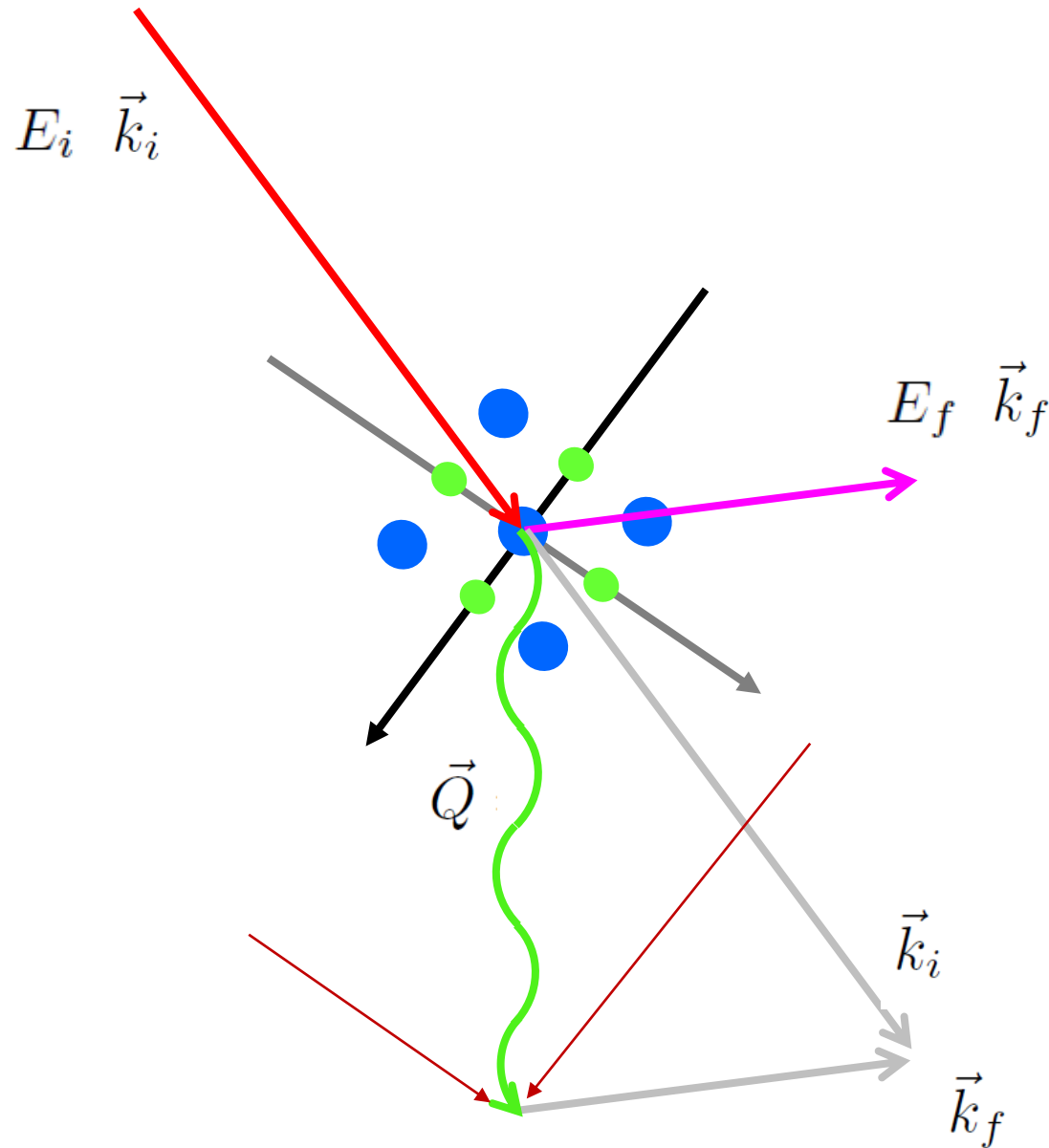
Single crystal



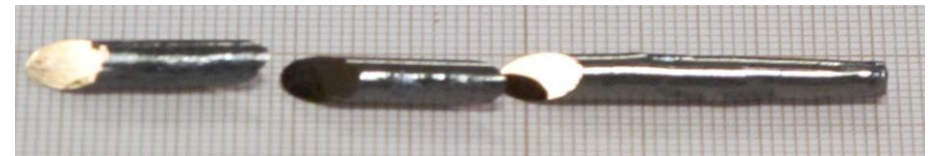
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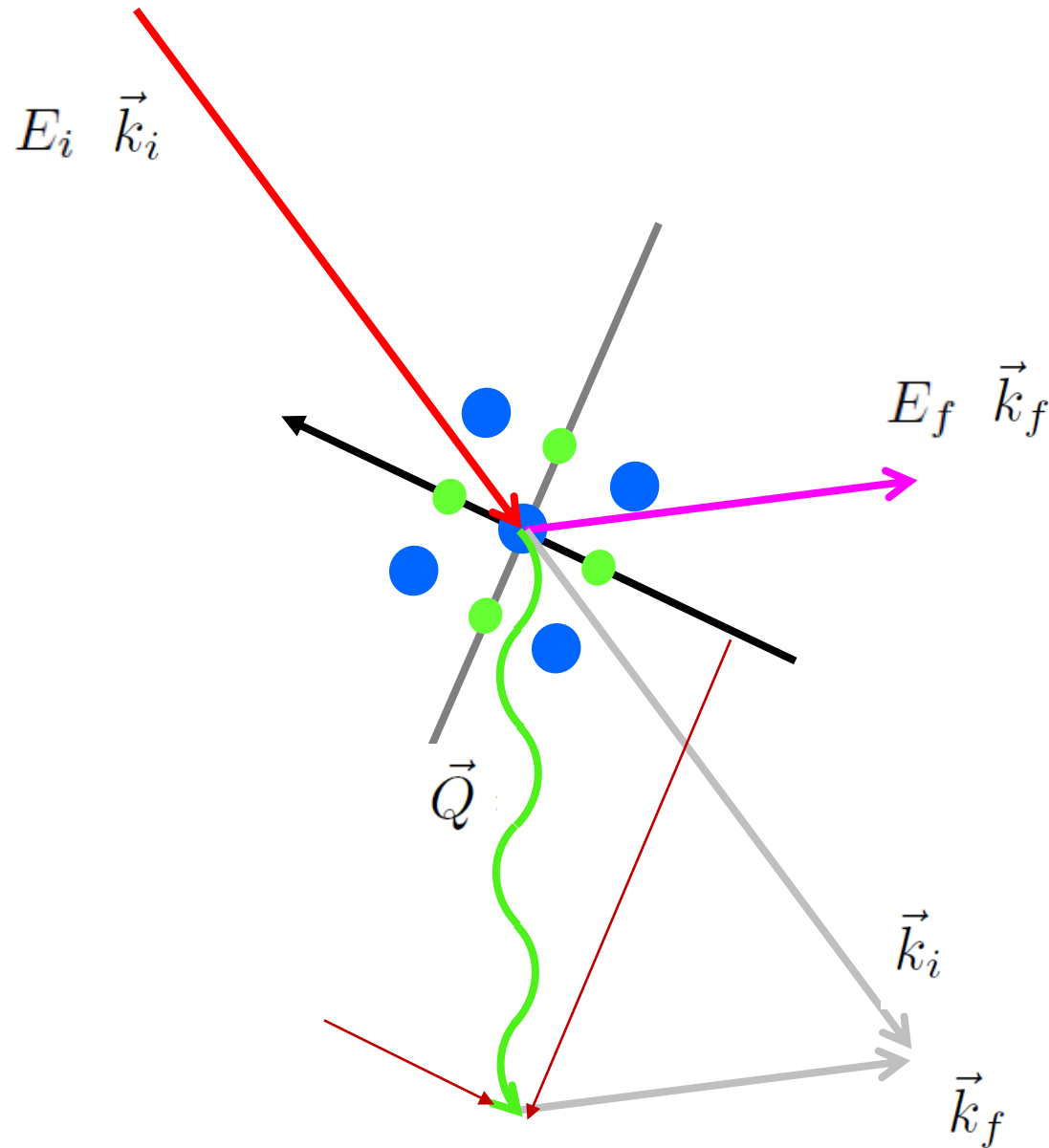
Single crystal



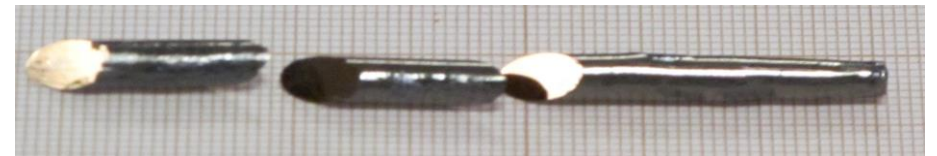
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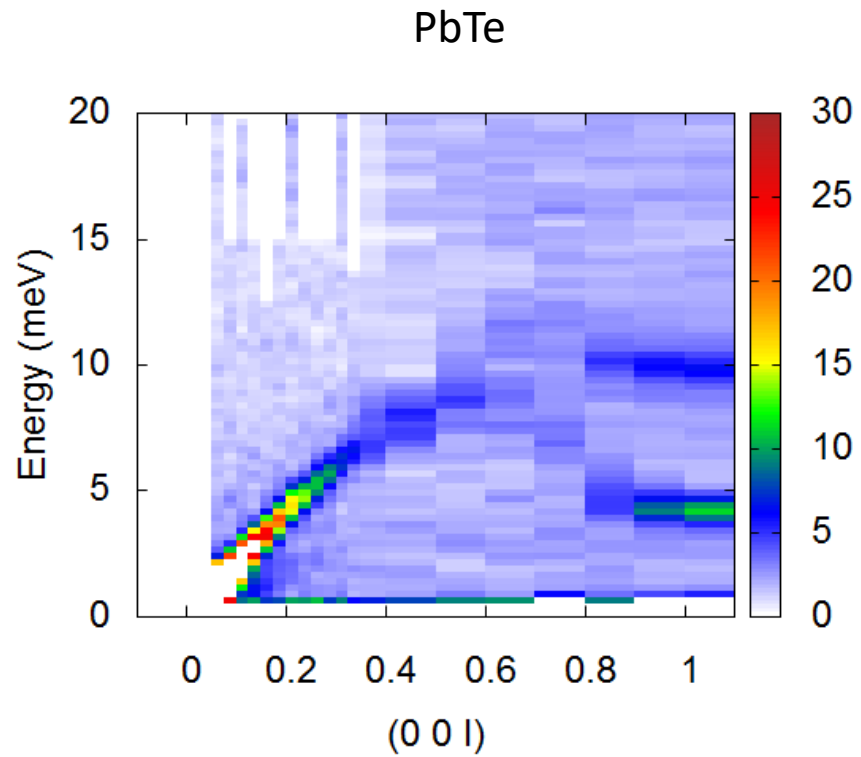


Single crystal



- Rotate the sample to select the components of $\vec{Q}$ with respect to the sample's lattice



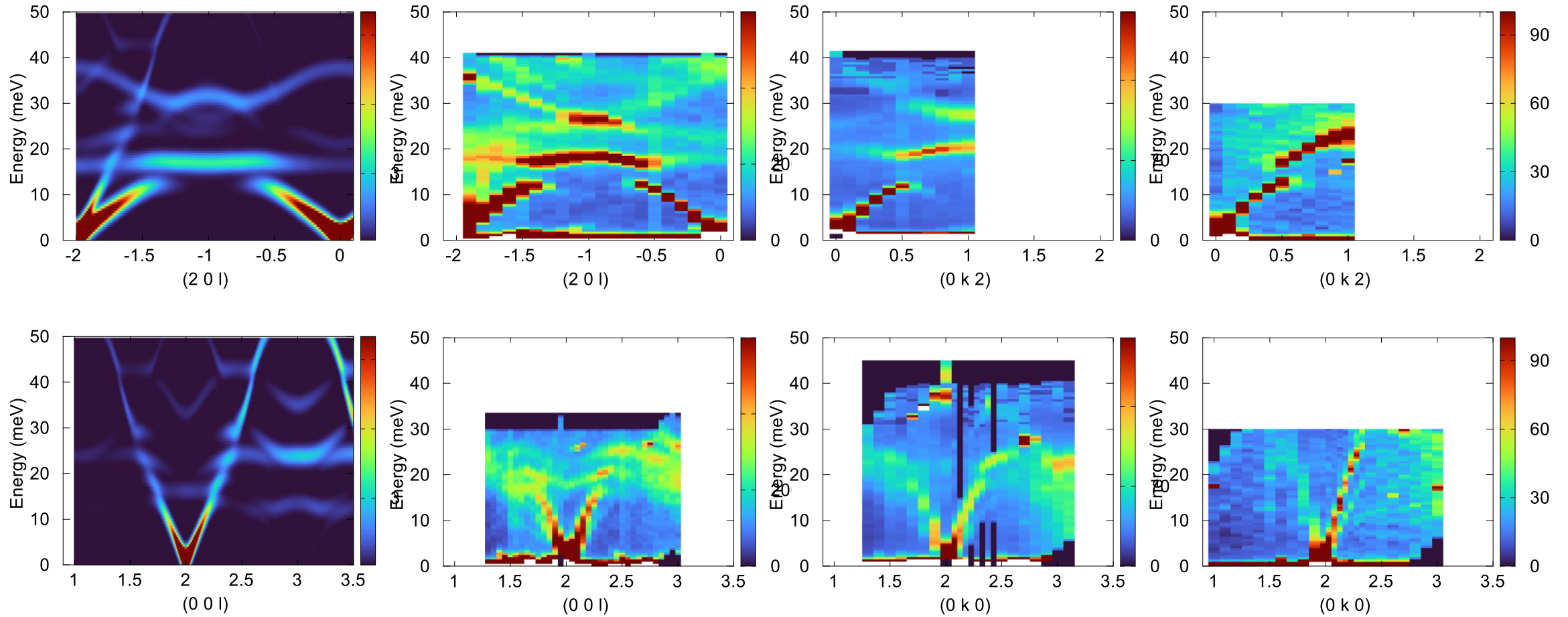


Direct observation of the phonon dispersion in (Q, ω) space !!

Q is a vector

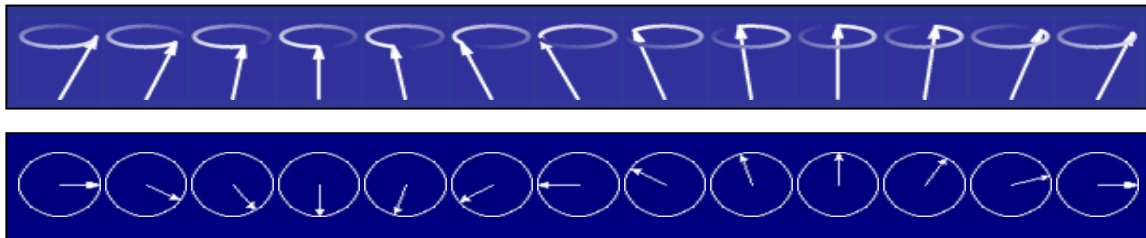
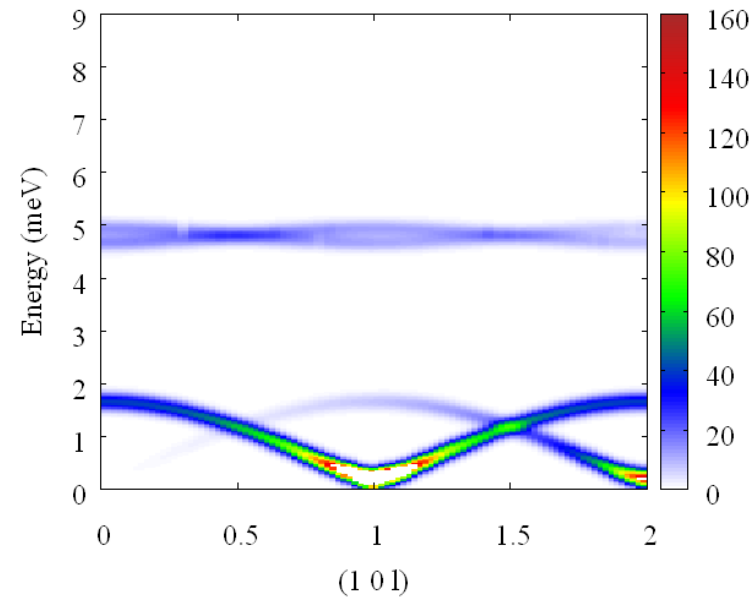
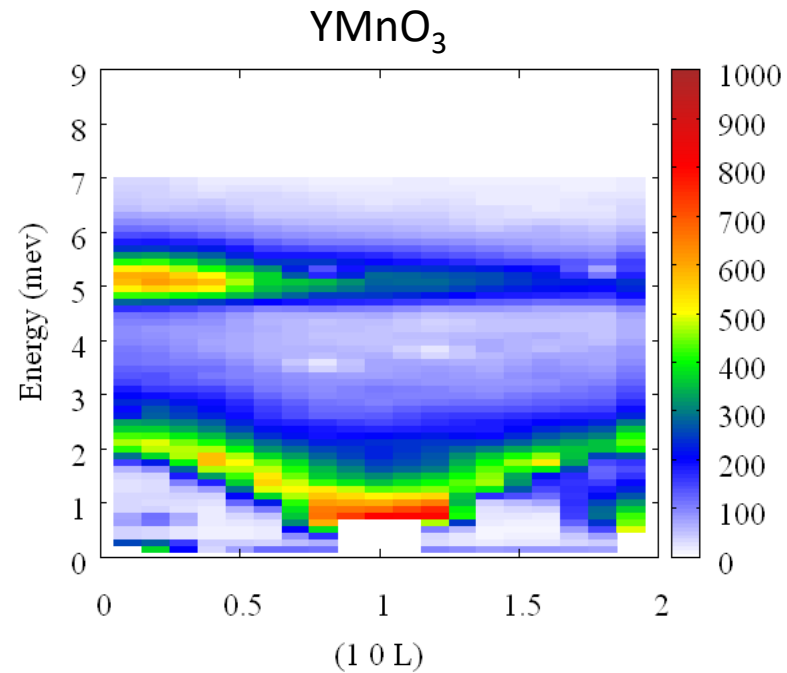
Phonons

SrCuO_2 , Sr_2CuO_3 , Ca_2CuO_3



Compare to DFT calculations and study the structure of materials

Spin waves



Direct observation of the spin wave dispersion in (Q, ω) space !!

Determine magnetic exchange and anisotropies

3. Cross sections



Inelastic scattering by phonons

$$\frac{\partial^2 \sigma}{\partial \Omega \partial E'} = \frac{k_f}{k_i} \sum_s |F_s(Q = \tau + k)|^2 \{ (1 + n(\omega)) \delta(\omega - \omega_{k,s}) + n(\omega) \delta(\omega + \omega_{k,s}) \}$$

Creation

Annihilation

- Phonons are bosons
- « detailed balance »
- Dynamical structure factor

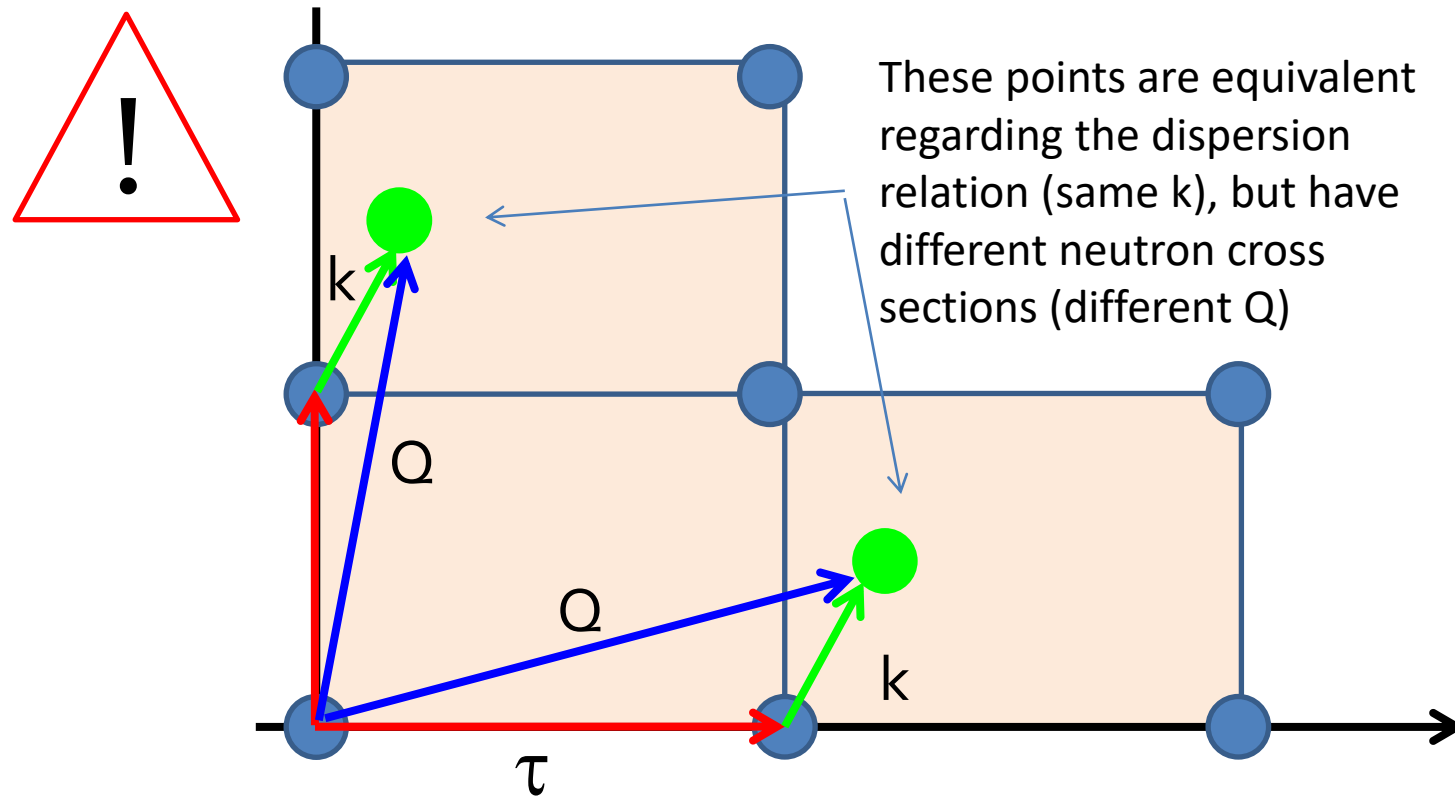
Bose-Einstein

$$n(\omega_{Q,s}) = \frac{1}{e^{\frac{\hbar \omega_{Q,s}}{k_B T}} - 1}$$

$$F_s(Q = \tau + k) = \sum_{\ell} b_{\ell} e^{i\vec{Q} \cdot \vec{r}_{\ell}} e^{-W_{\ell}} \frac{1}{\sqrt{M_{\ell} \omega_{k,s}}} (\vec{Q} \cdot \vec{e}_{k,\ell}^{(s)})$$

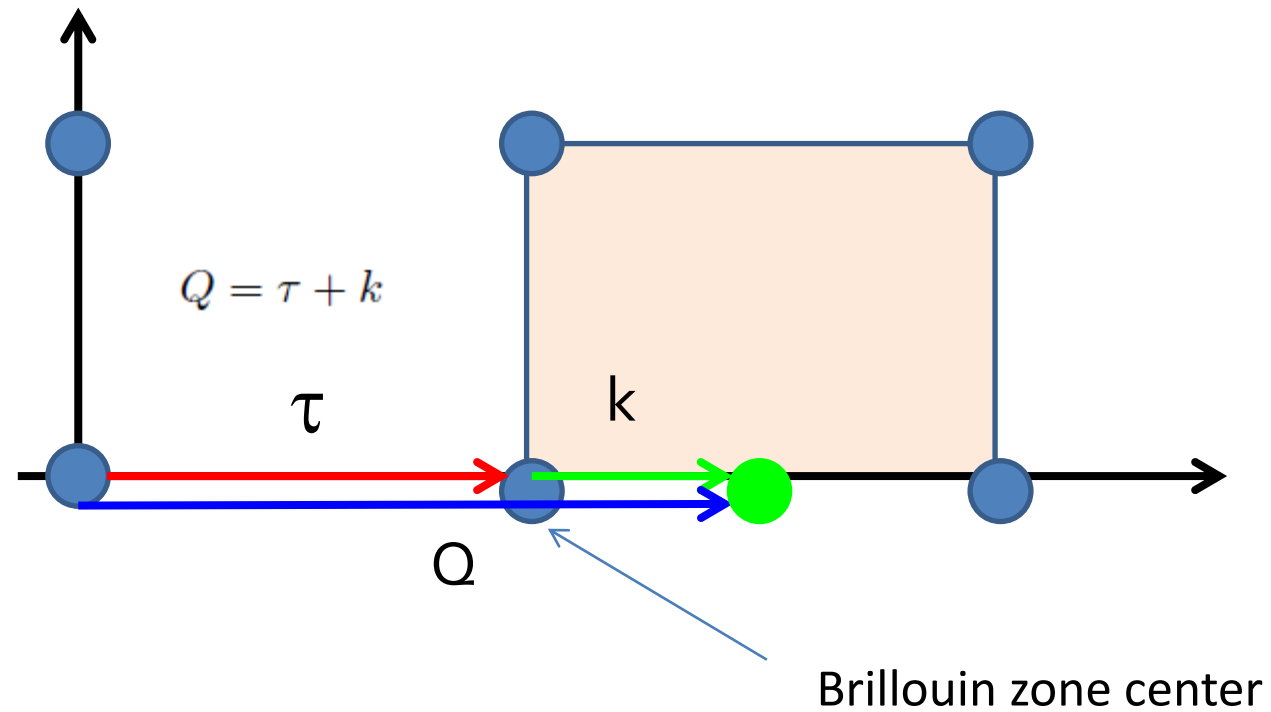
$\omega(k)$ is periodic. It suffices to study it in a single Brillouin zone

$$F_s(Q = \tau + k) = \sum_{\ell} b_{\ell} e^{i\vec{Q} \cdot \vec{r}_{\ell}} e^{-W_{\ell}} \frac{1}{\sqrt{M_{\ell} \omega_{k,s}}} (\vec{Q} \cdot \vec{e}_{k,\ell}^{(s)})$$



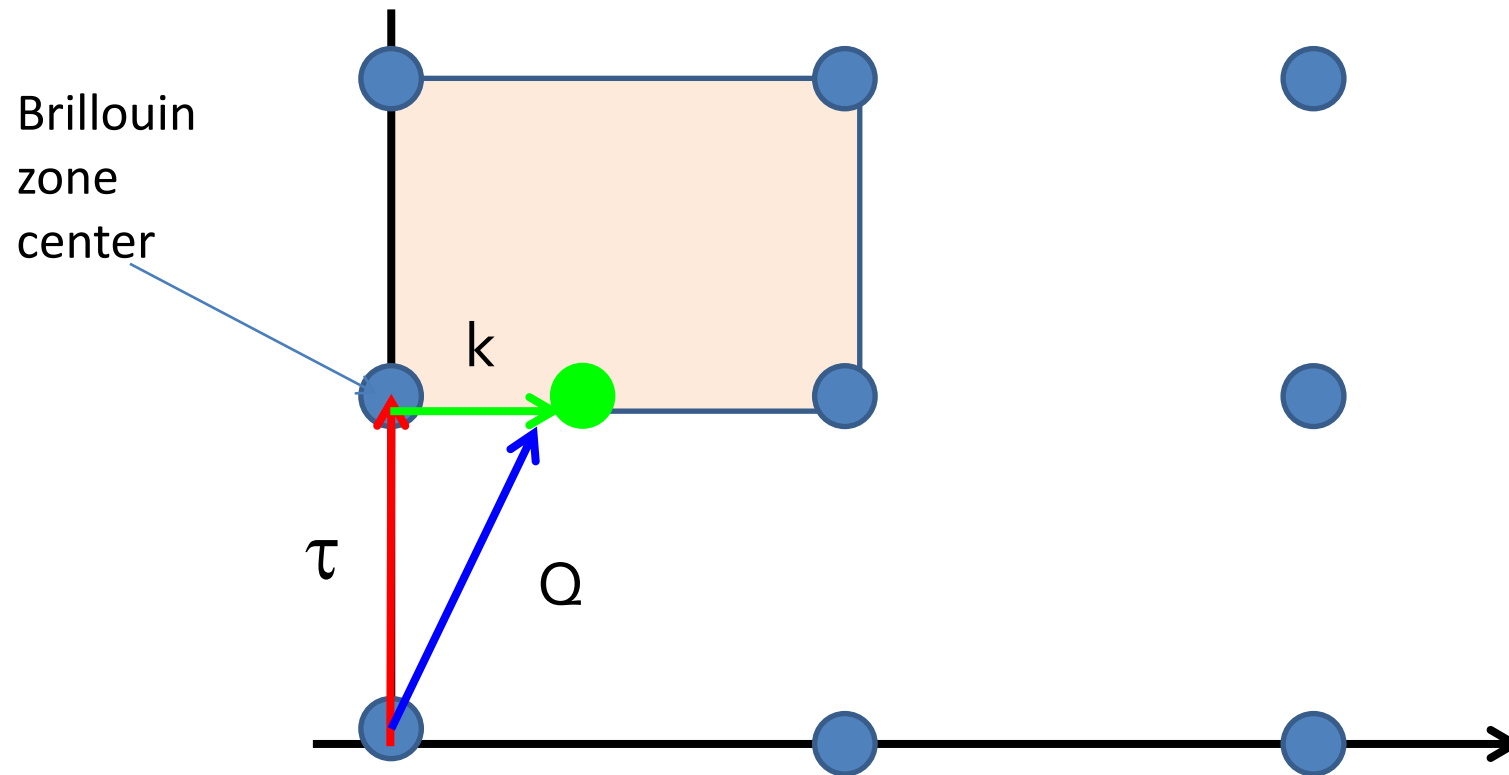
τ = Brillouin zone node = Bragg peak position
 k = wavevector within the Brillouin zone

$$F_s(Q = \tau + k) = \sum_{\ell} b_{\ell} e^{i\vec{Q} \cdot \vec{r}_{\ell}} e^{-W_{\ell}} \frac{1}{\sqrt{M_{\ell}} \omega_{k,s}} (\vec{Q} \cdot \vec{e}_{k,\ell}^{(s)})$$



Longitudinal phonon propagating along x :

$$F_s(Q = \tau + k) = \sum_{\ell} b_{\ell} e^{i\vec{Q} \cdot \vec{r}_{\ell}} e^{-W_{\ell}} \frac{1}{\sqrt{M_{\ell}} \omega_{k,s}} (\vec{Q} \cdot \vec{e}_{k,\ell}^{(s)})$$



Transverse phonon propagating along x

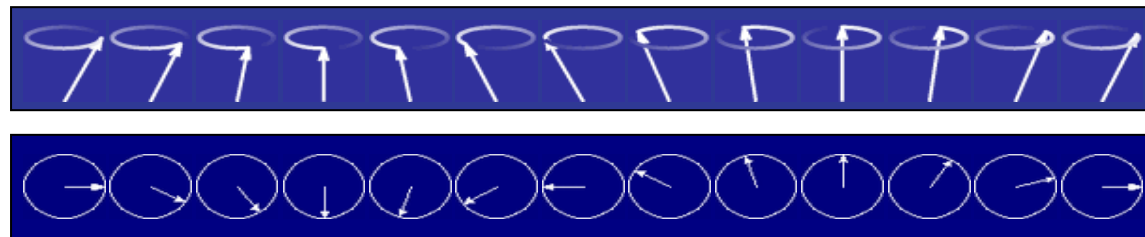
Inelastic scattering by spin waves

$$\frac{\partial^2 \sigma}{\partial \Omega \partial E'} = \sum_s A_s [(1 + n(\omega_{Q,s})) \delta(\omega - \omega_{Q,s}) + n(\omega_{Q,s}) \delta(\omega + \omega_{Q,s})]$$

Creation

Annihilation

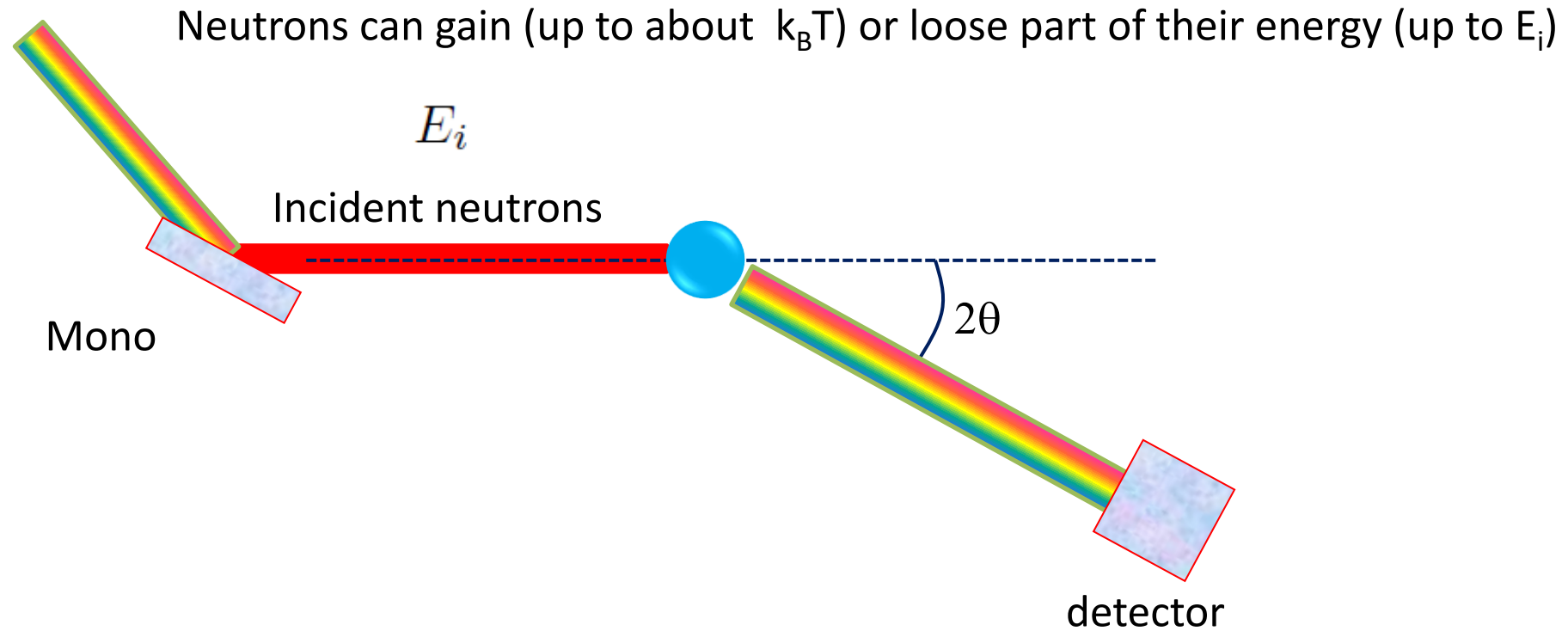
- Spin waves are bosons
- « detailed balance »



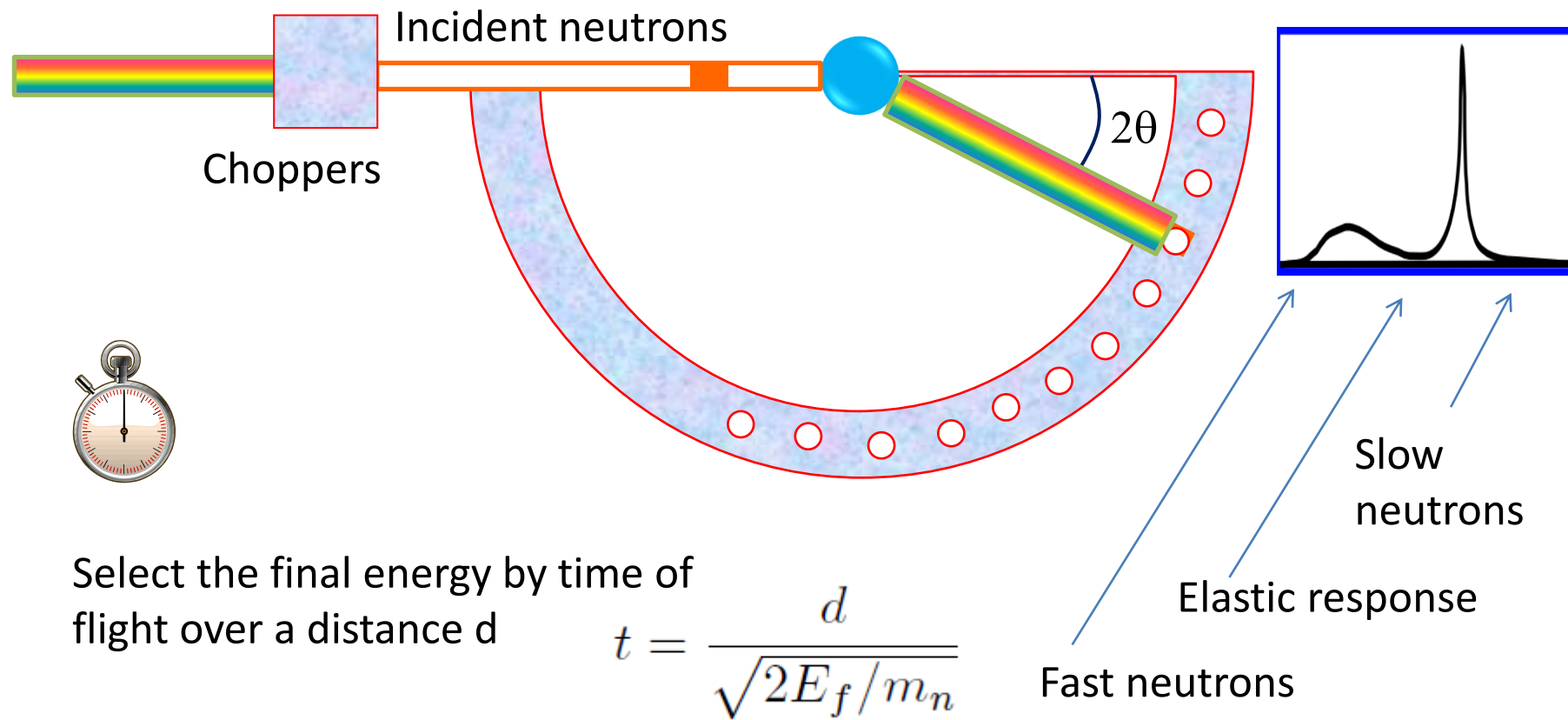
4. TOF



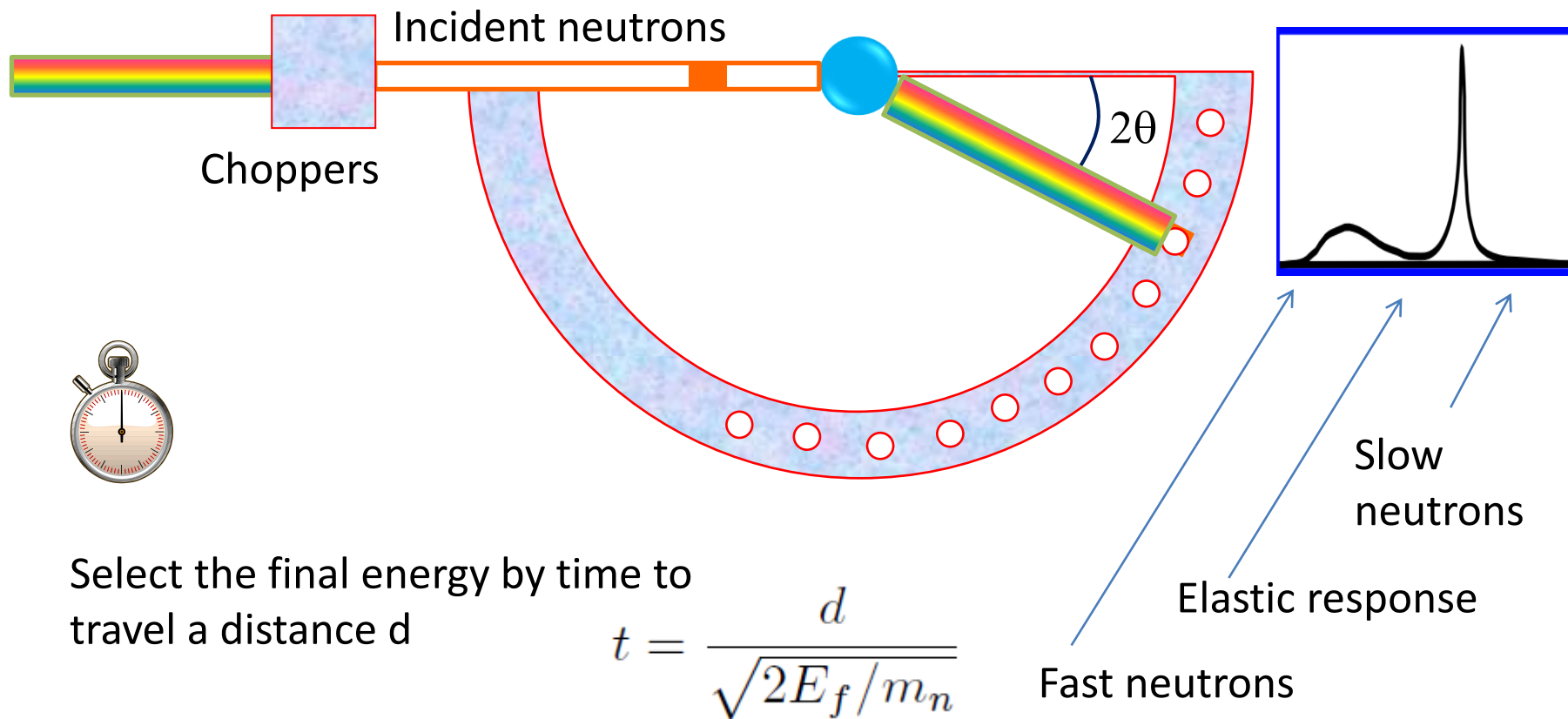
How to perform INS ?



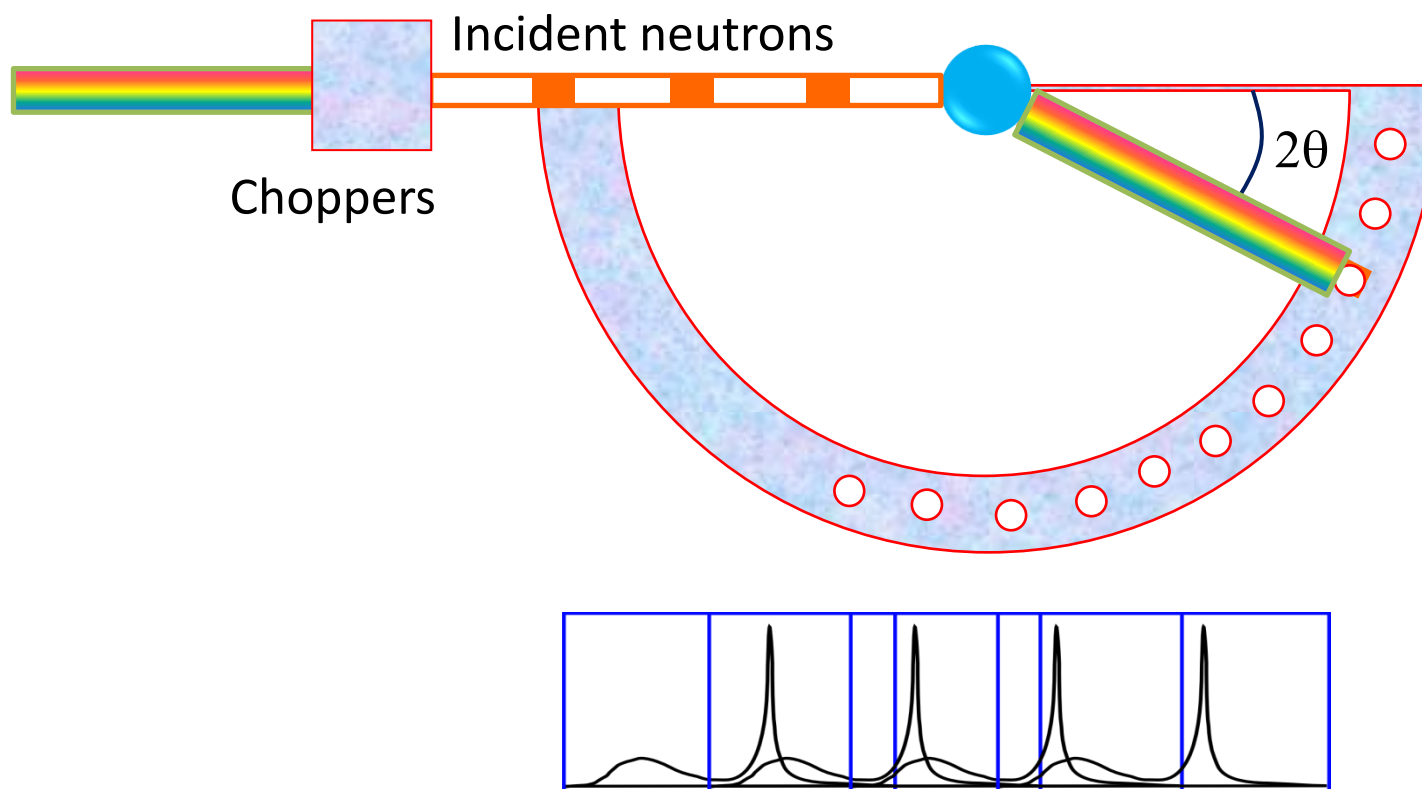
How to determine E_f ?



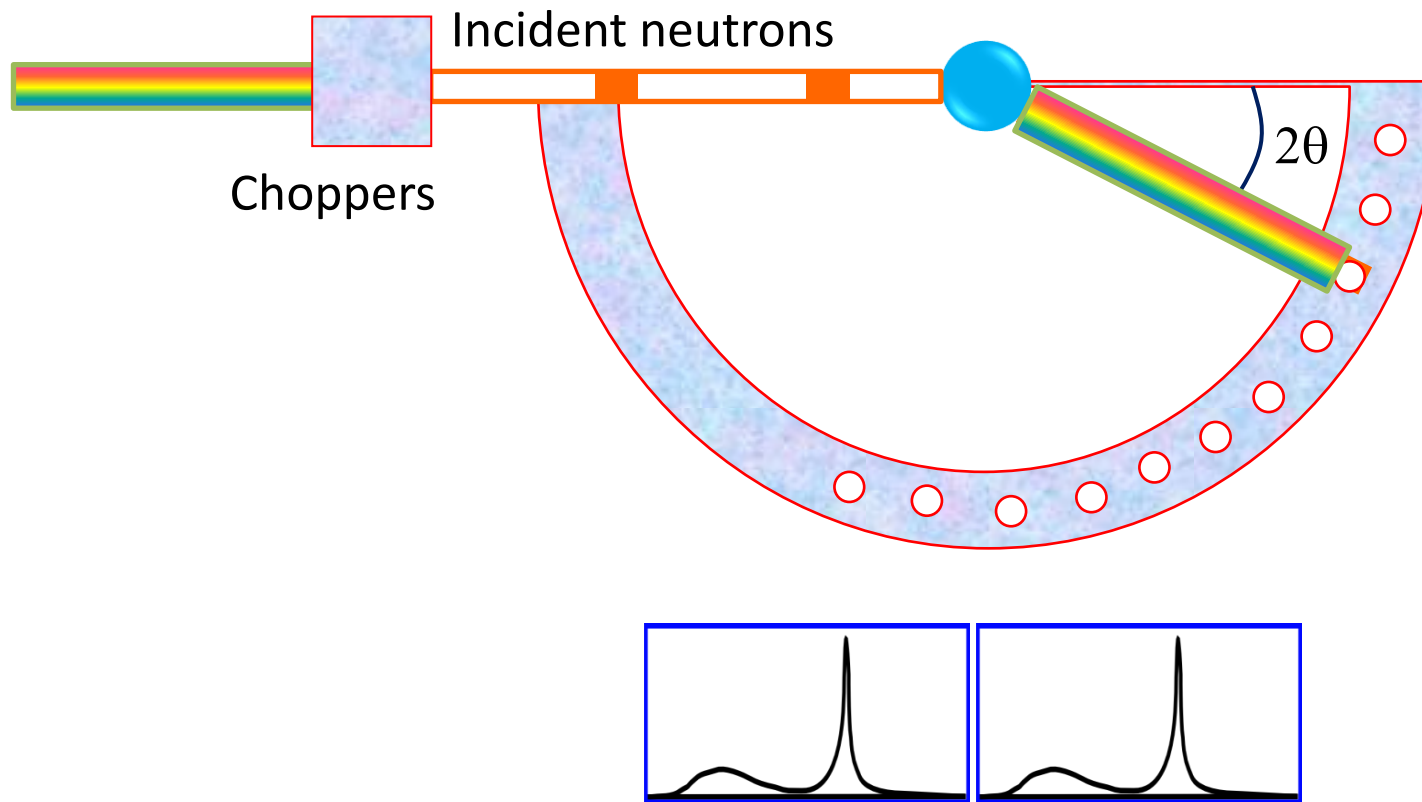
GREAT !! Measures all 2θ s (hence $|Q|$) at the same time

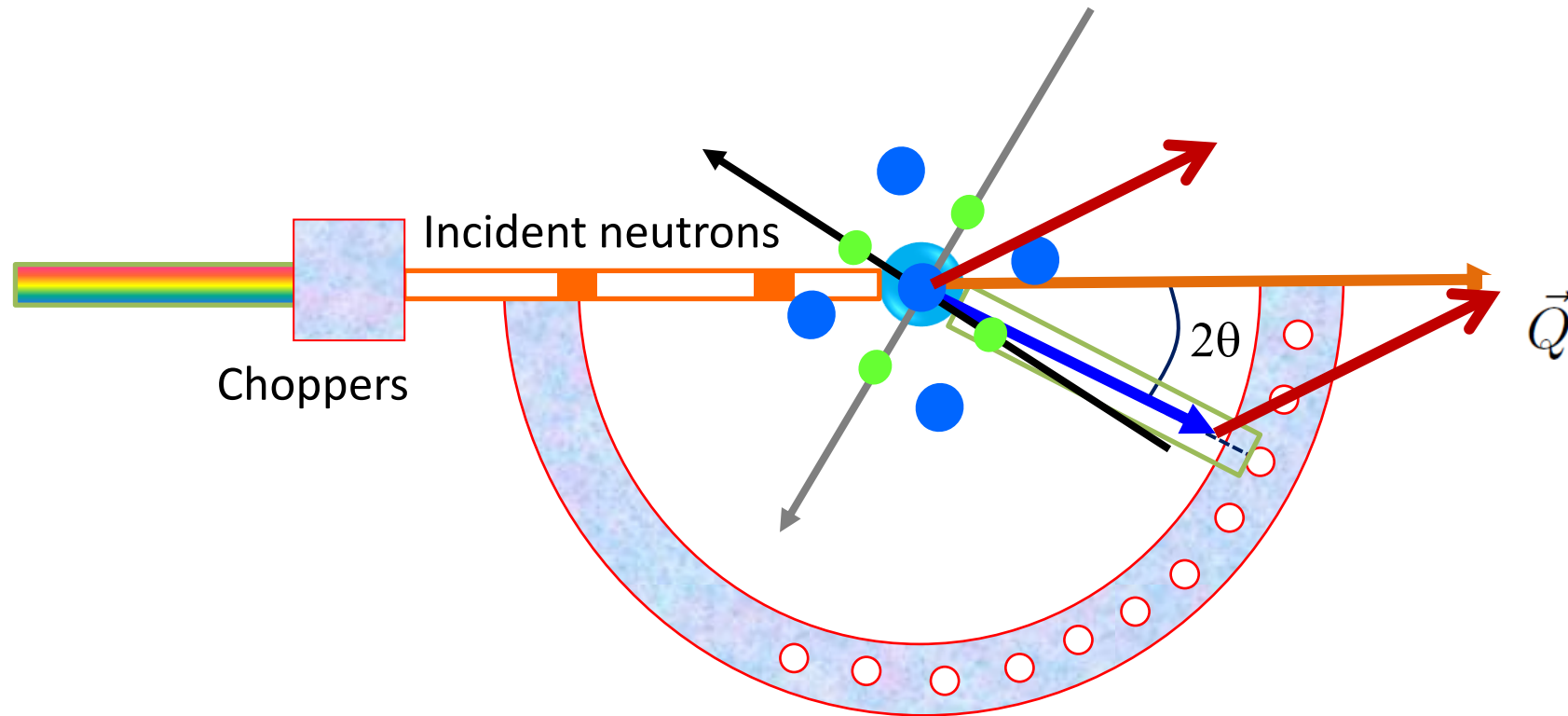


BUT Frame overlap ...

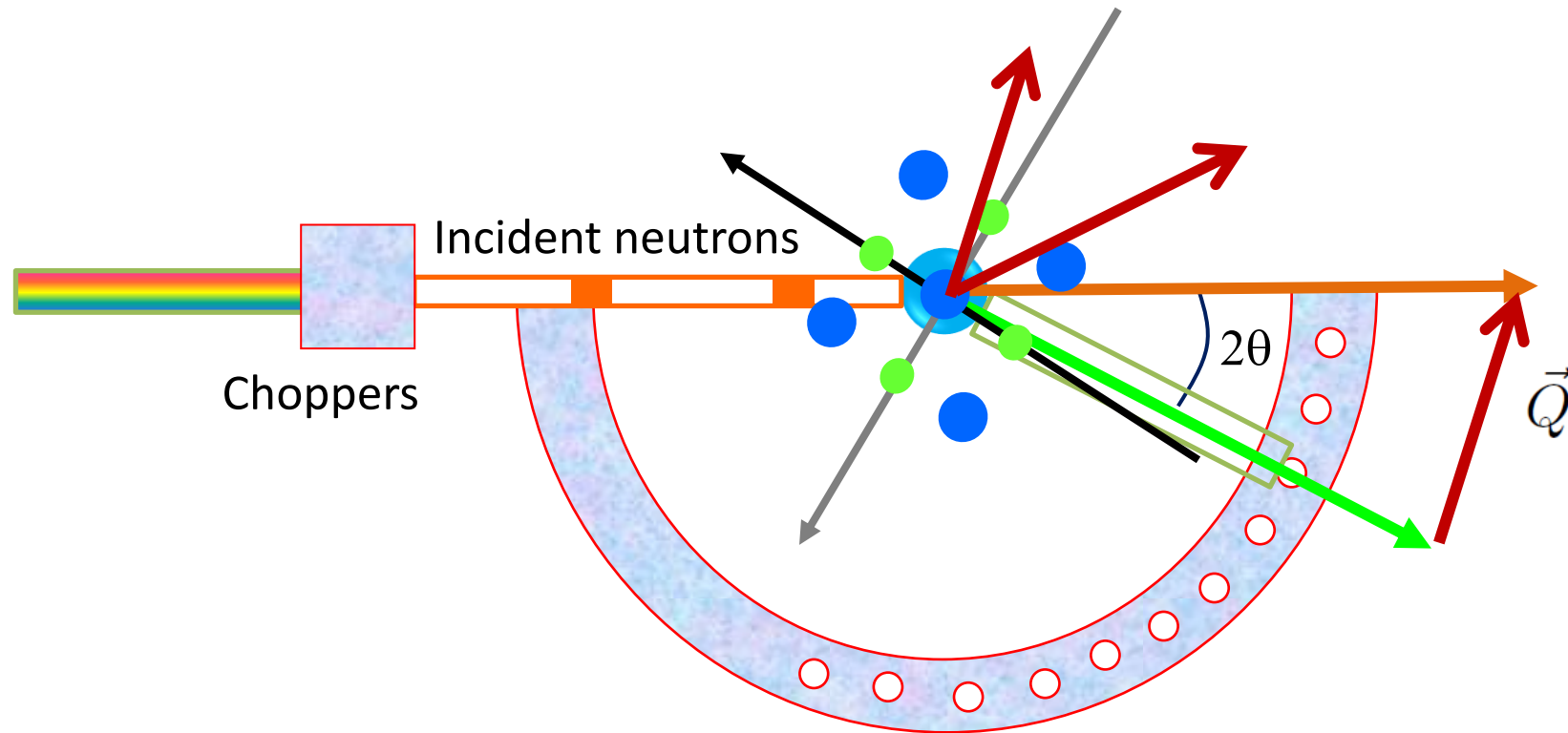


BUT Frame overlap, chop the beam (reduce the flux)

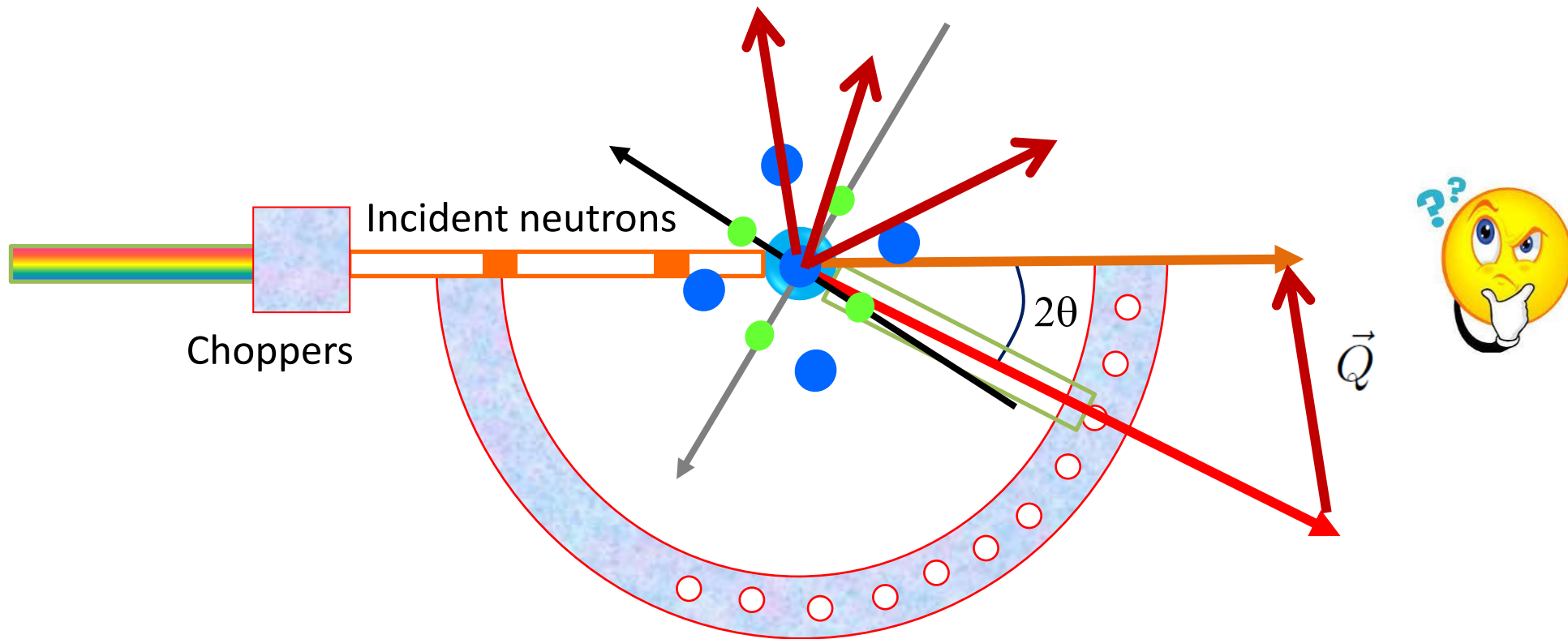




Sample rotation : for a given 2θ , at time t (hence given final energy), the scattering is due to different Q s (modulus and components)



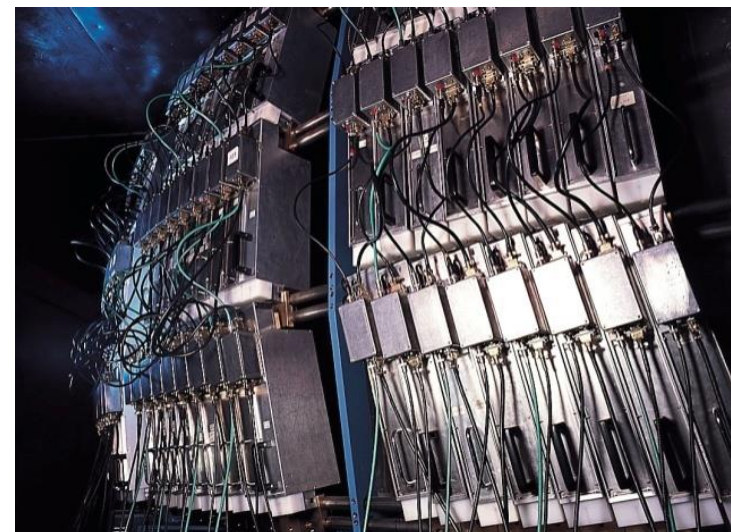
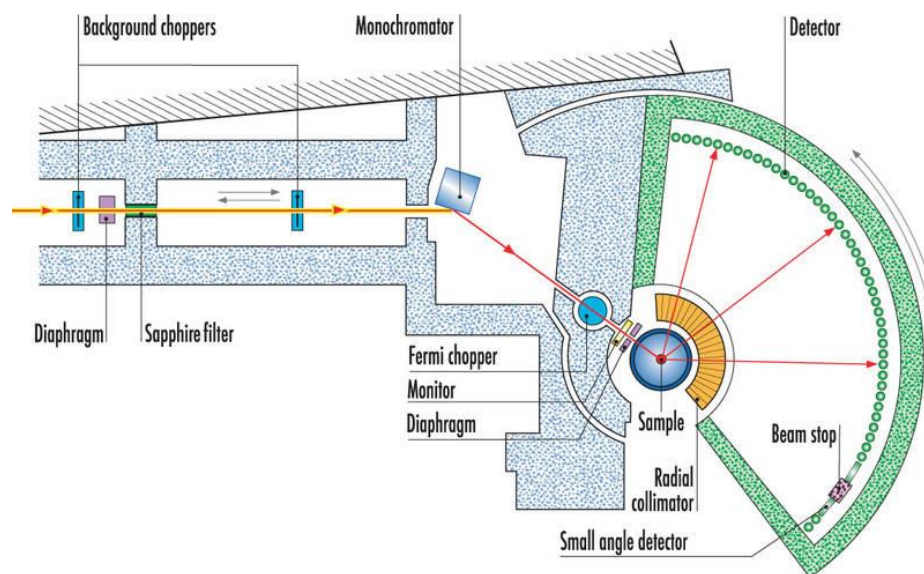
Sample rotation : for a given 2θ , at time t (hence given final energy), the scattering is due to different Q s (modulus and components)



Sample rotation : for a given 2θ , different (Q_s , E_f) are probed, as a function of time of flight: **it is necessary to repeat for a number of orientations of the sample**

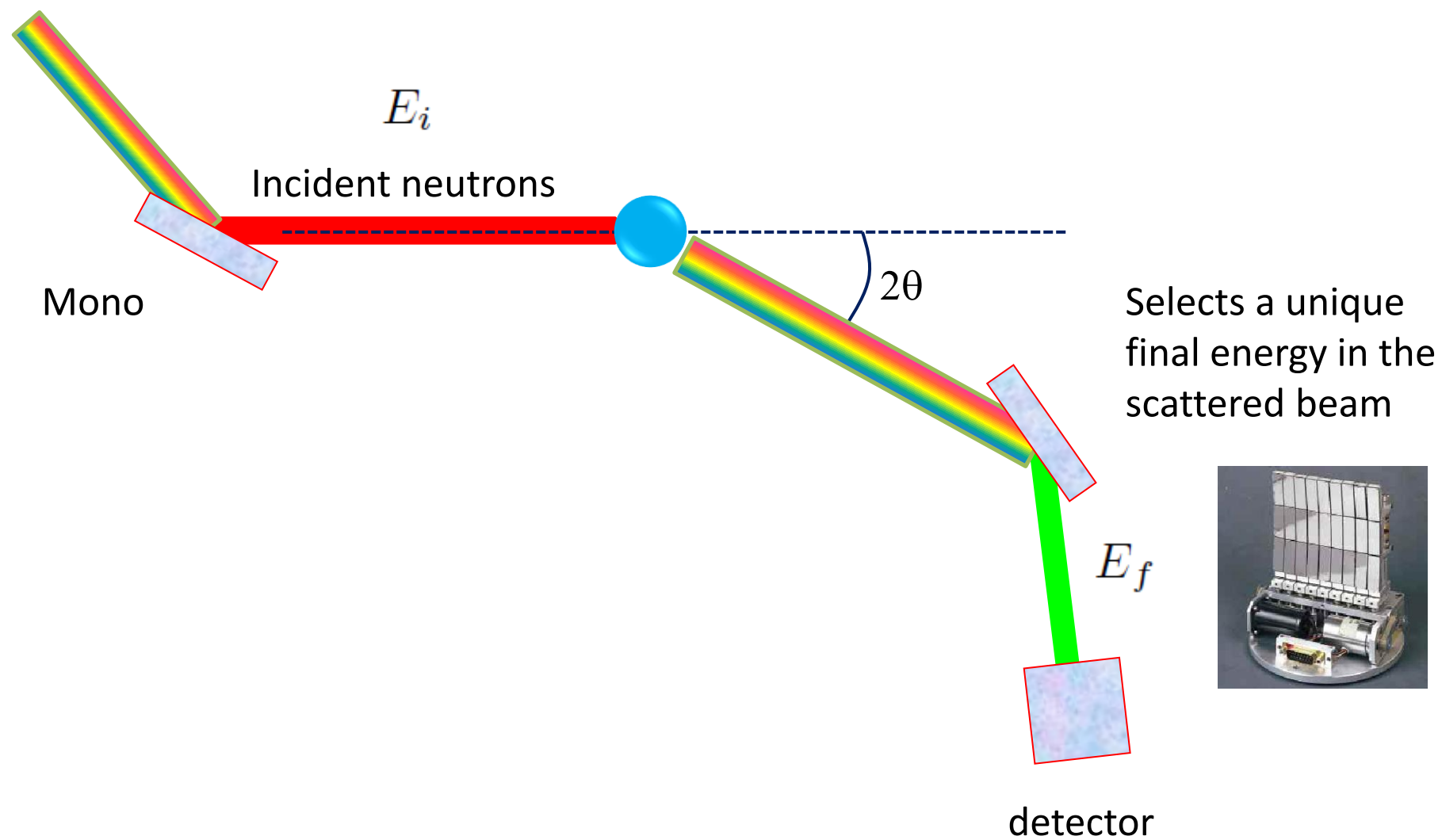
Time of flight spectrometer
IN4, IN5, IN6-Sharp @ ILL
FOCUS @ PSI

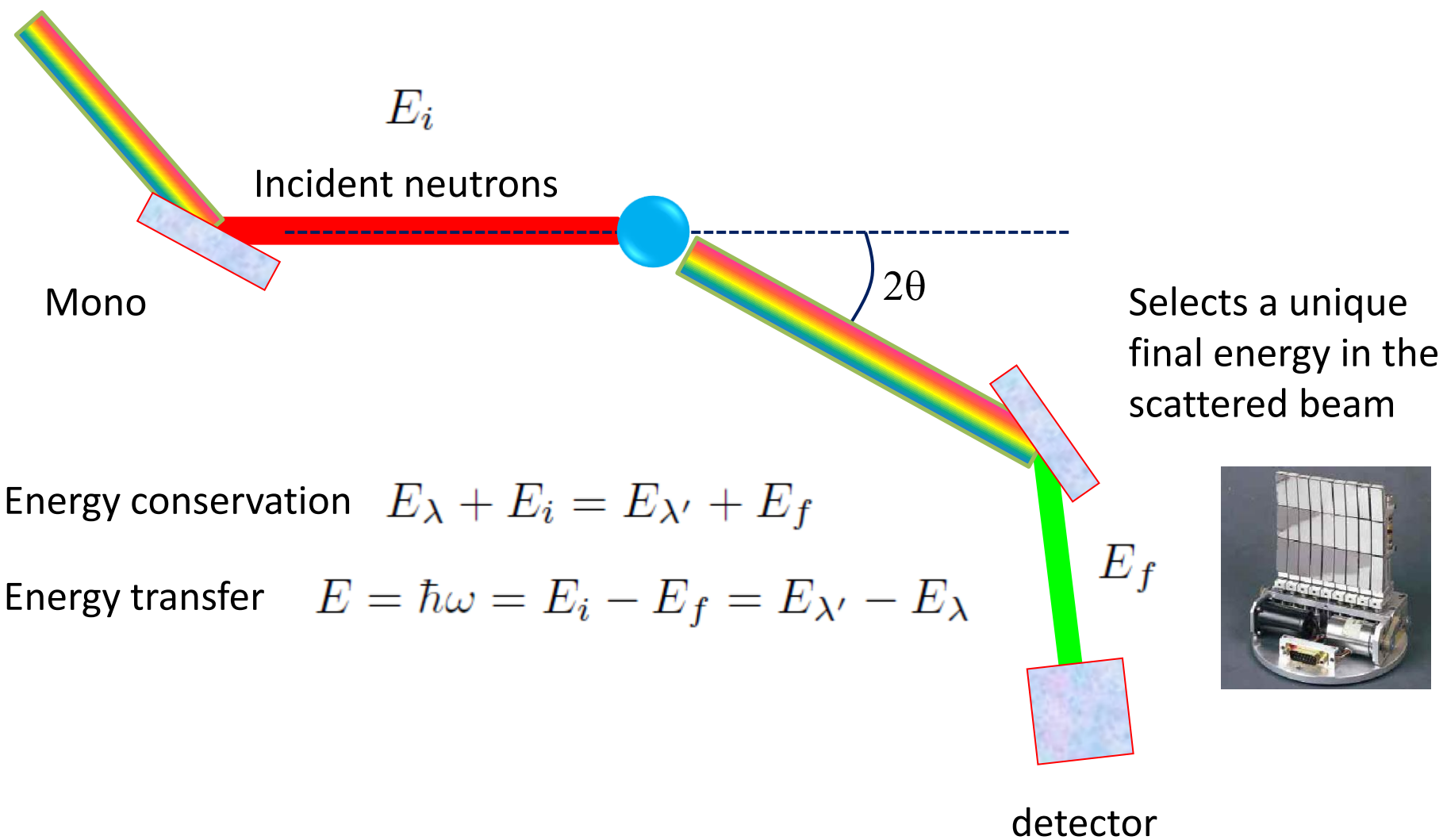
...

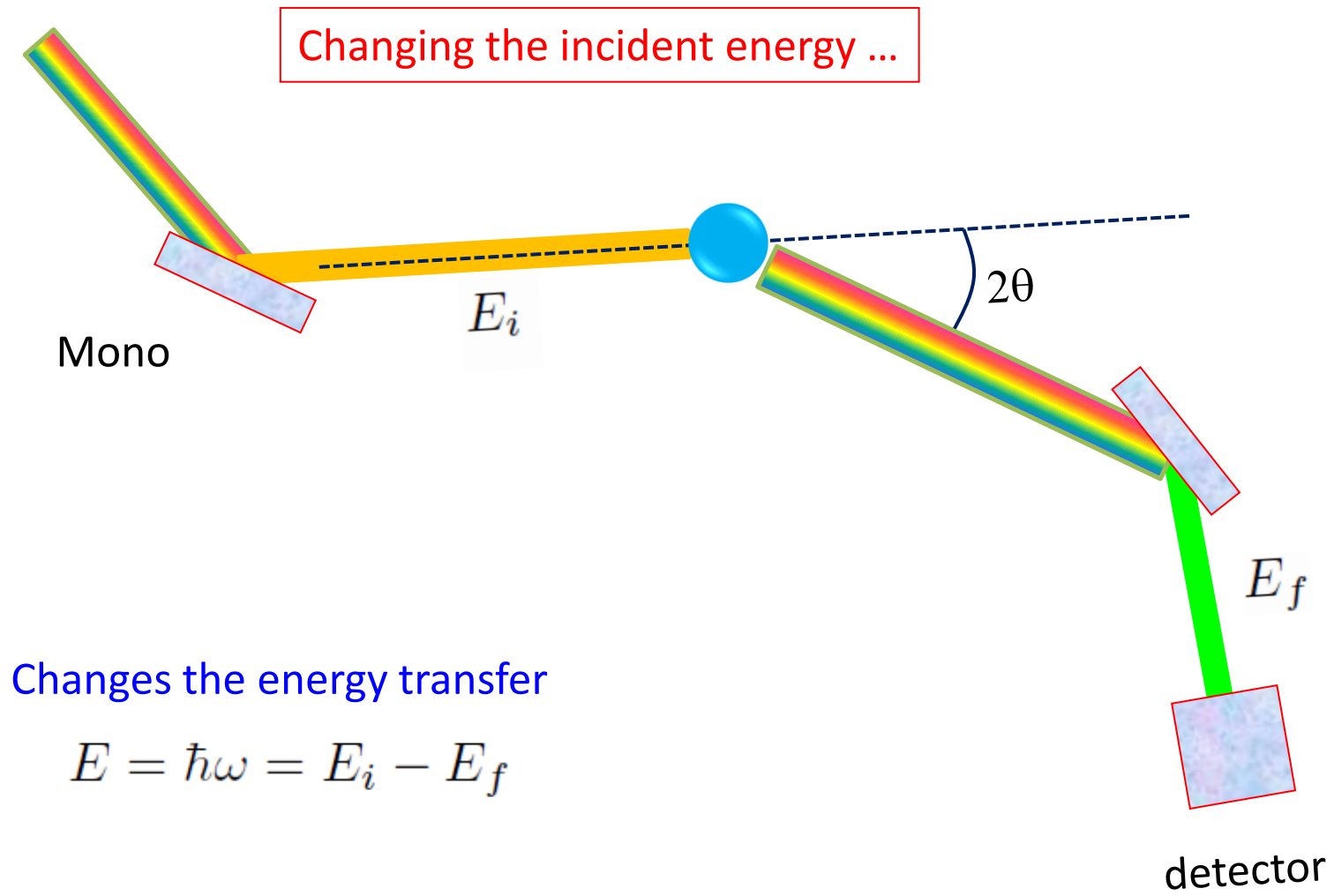


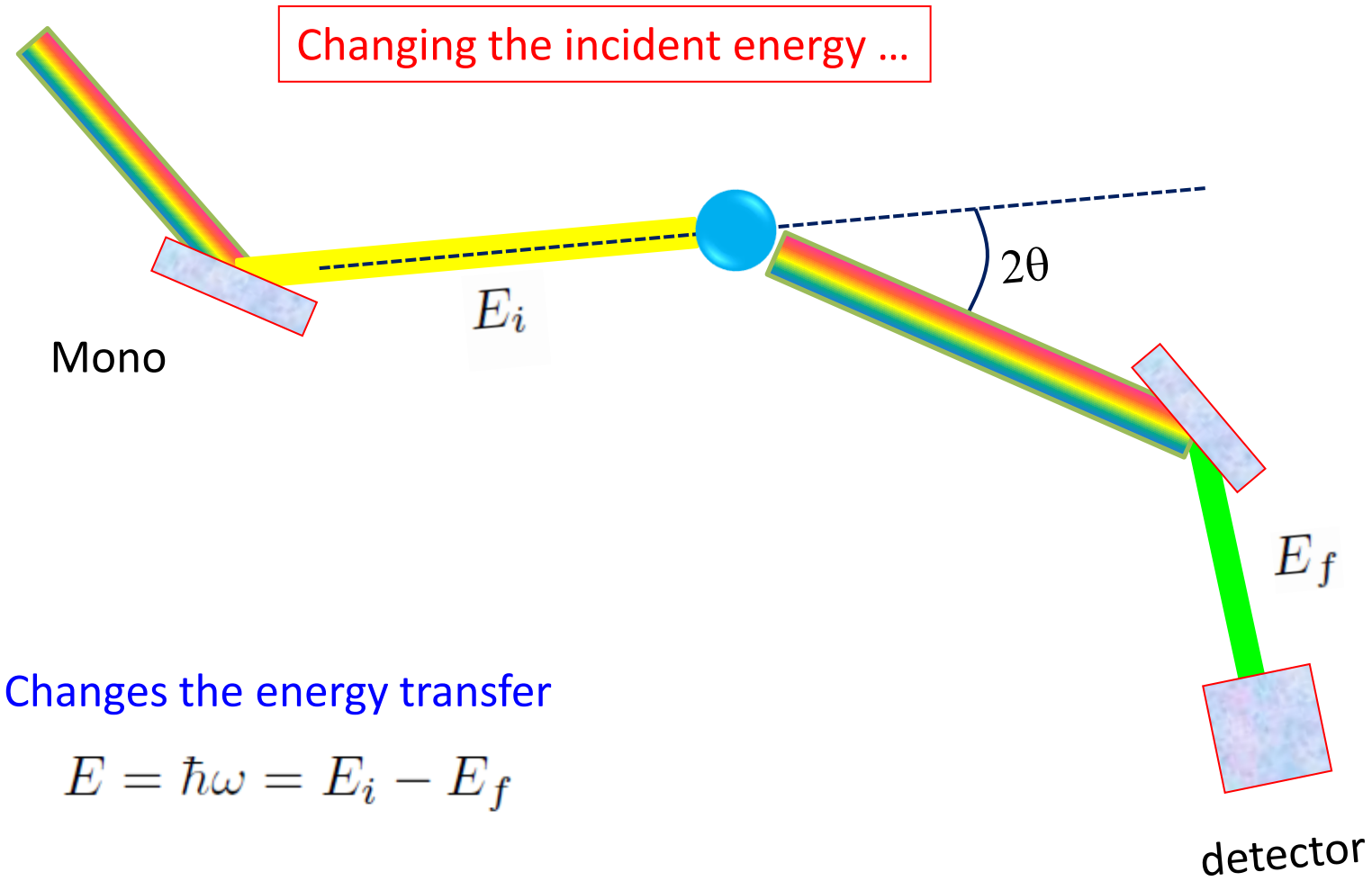
5. TAS

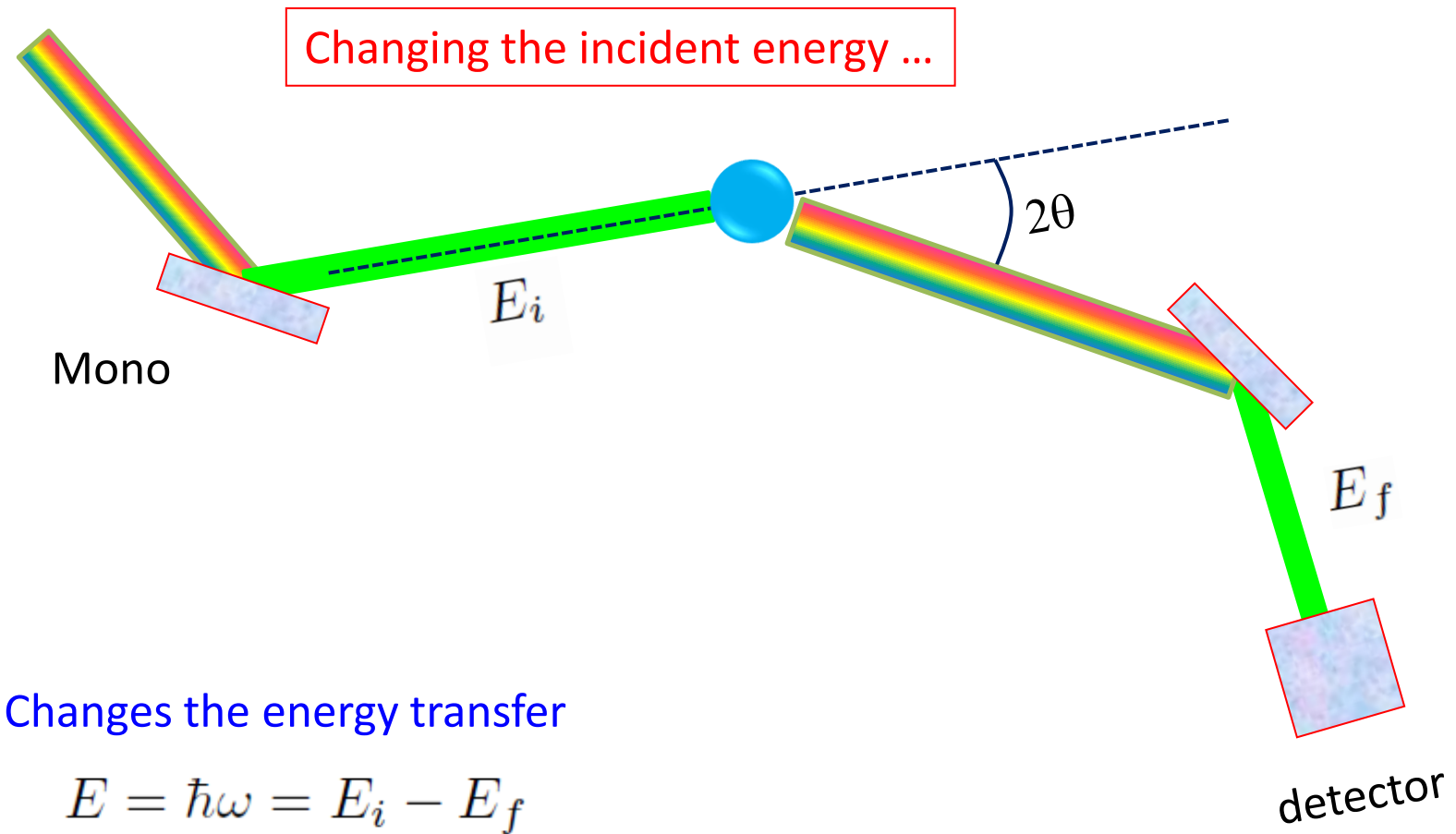


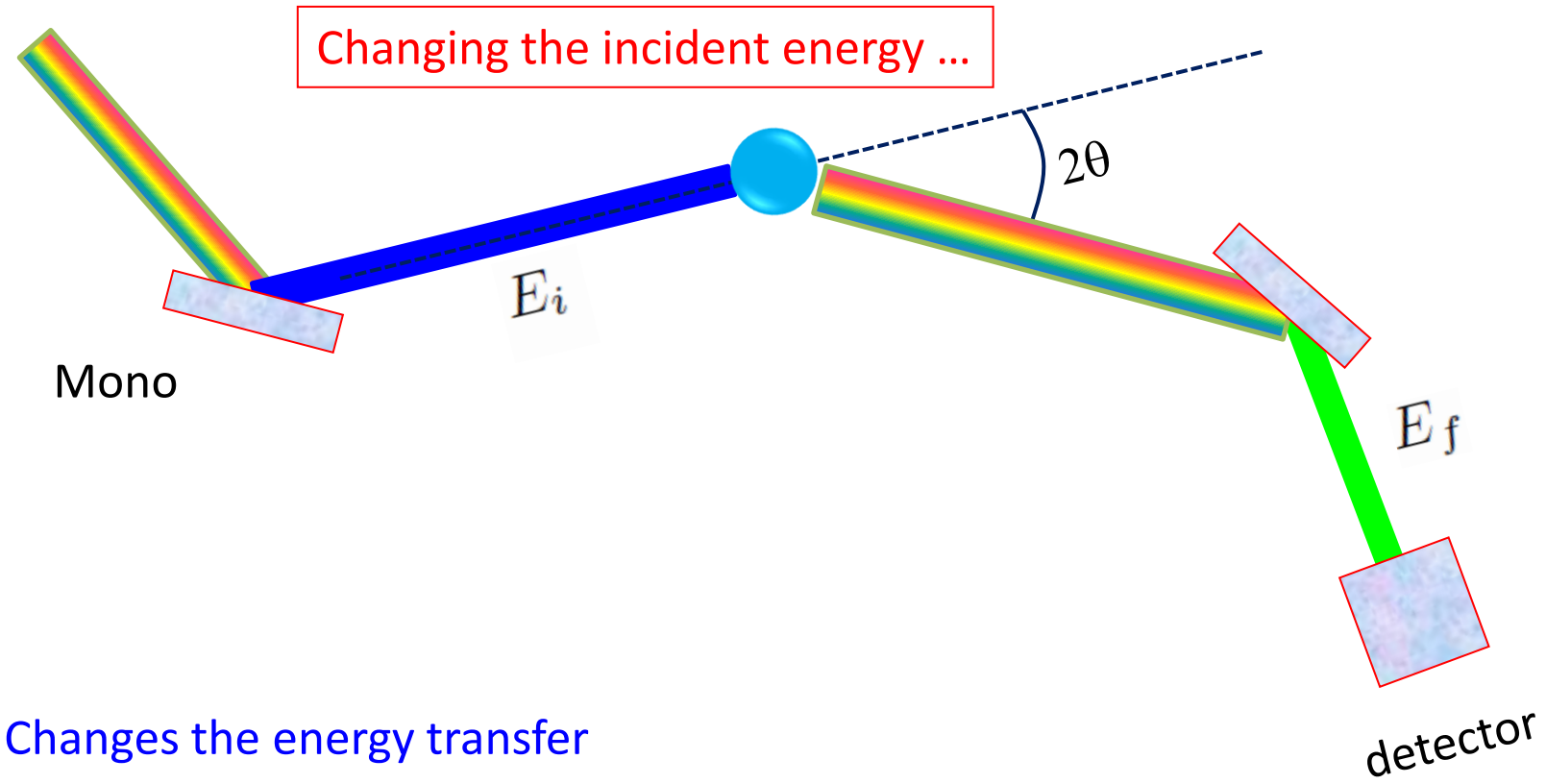






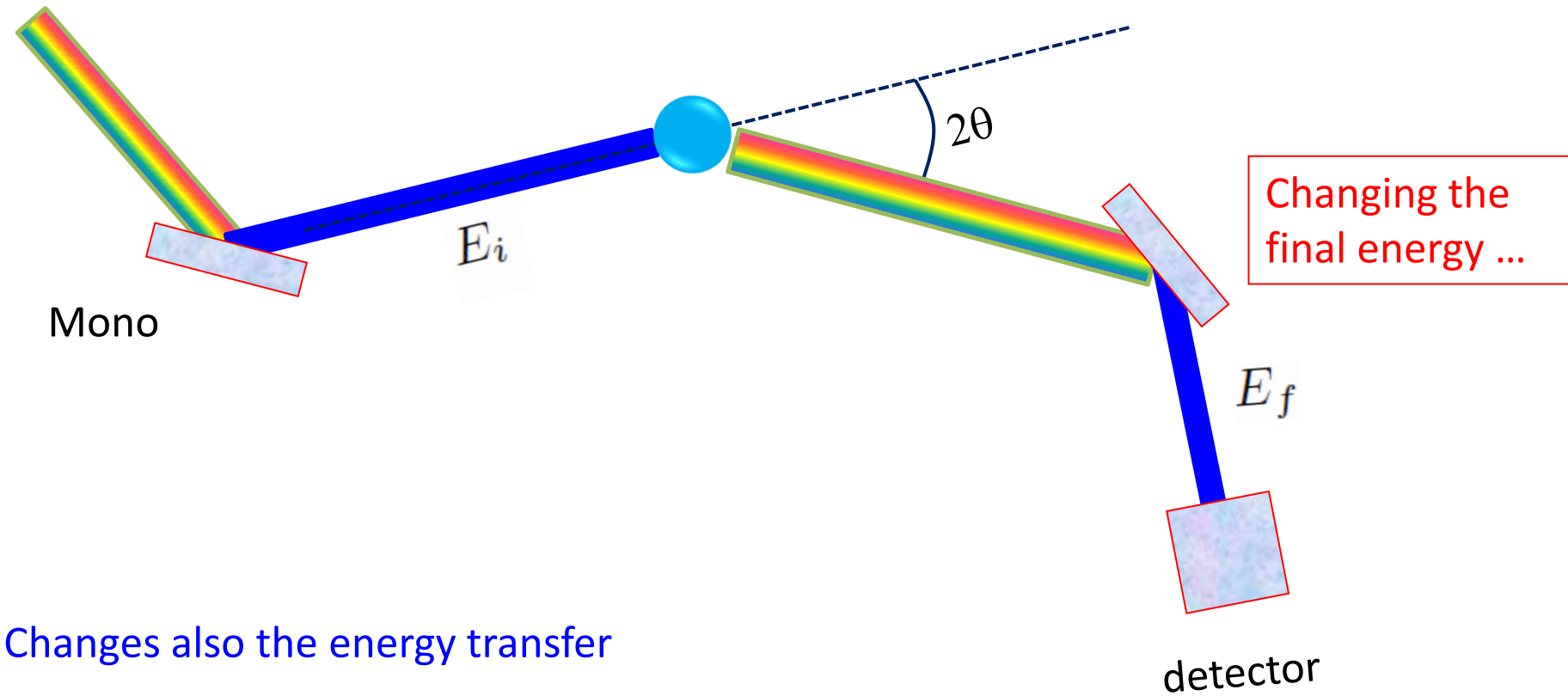






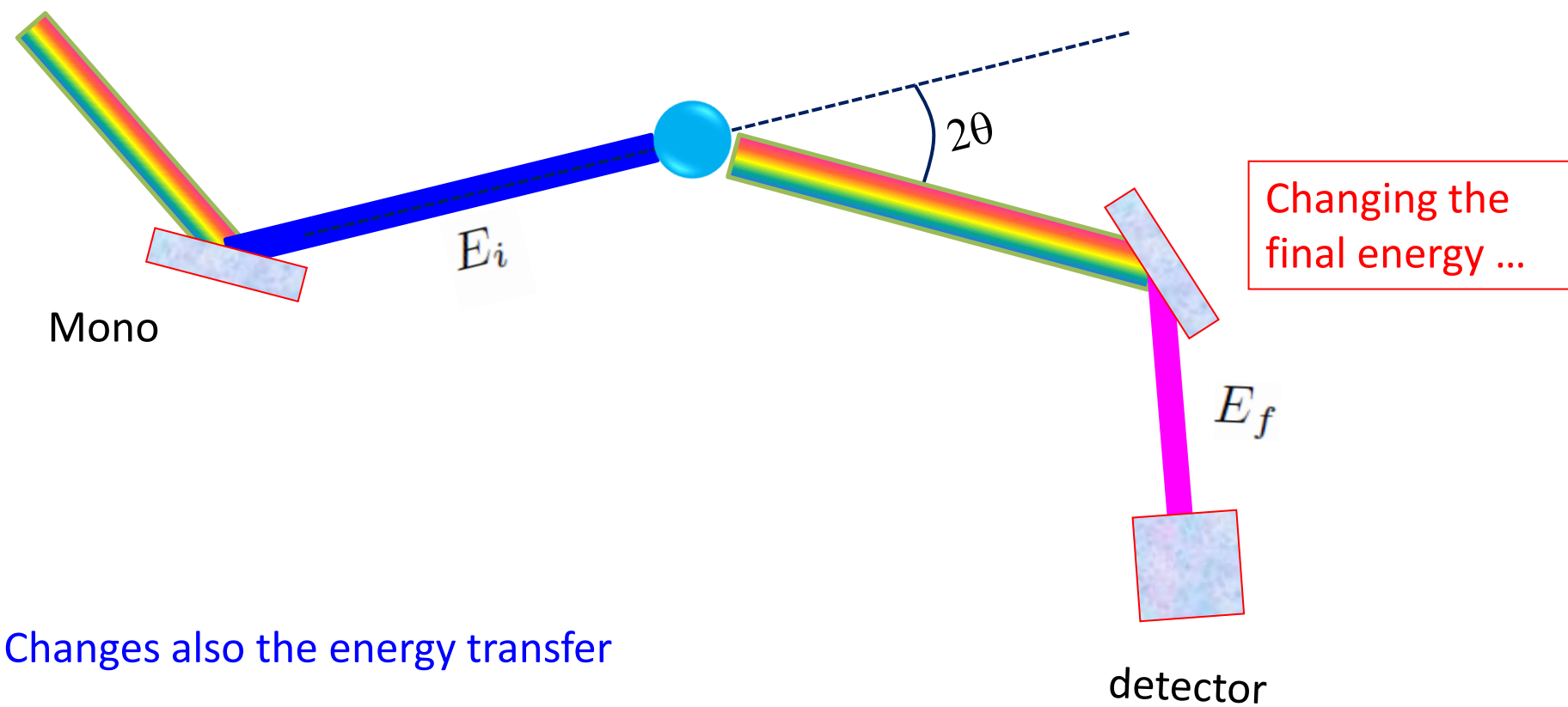
Changes the energy transfer

$$E = \hbar\omega = E_i - E_f$$



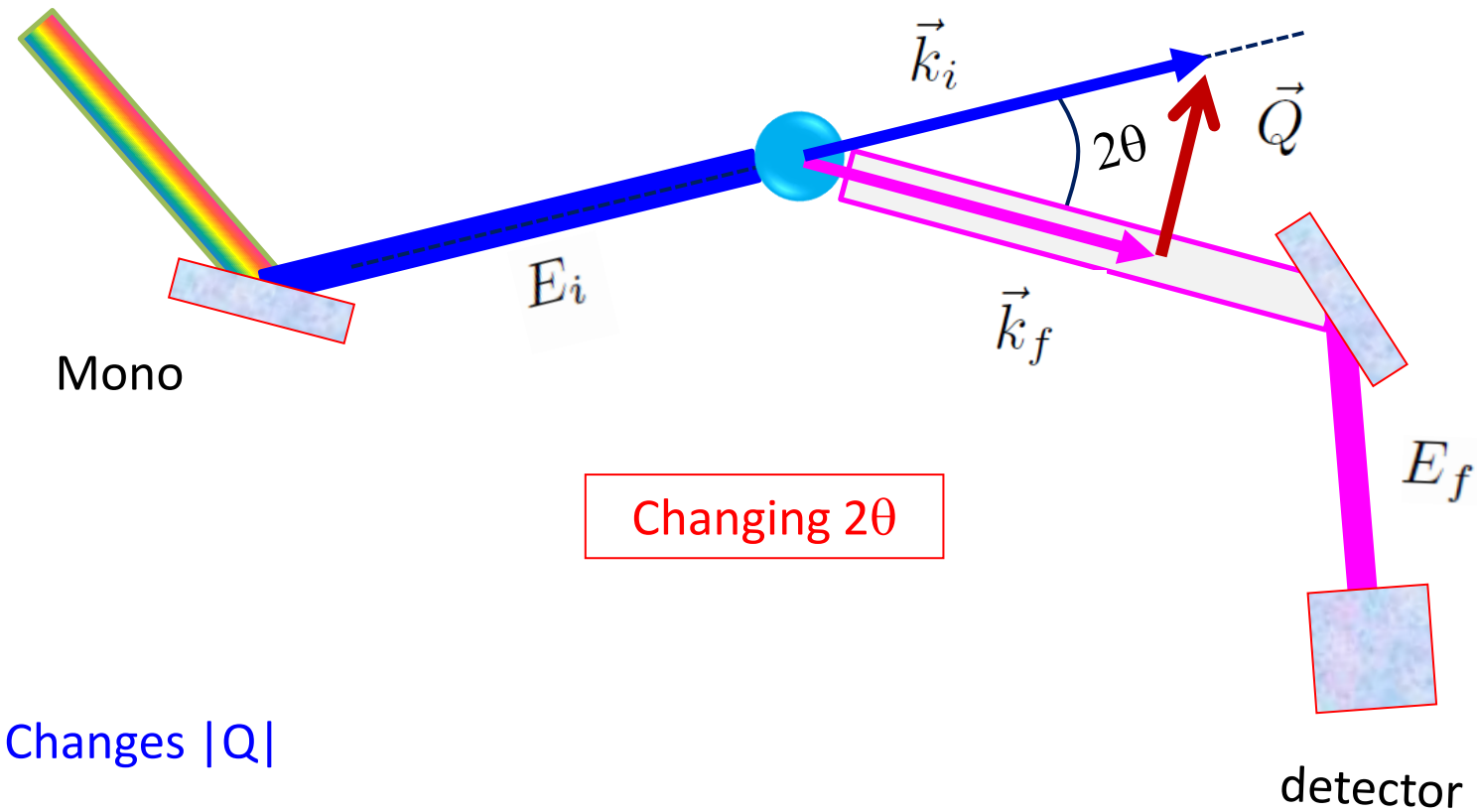
Changes also the energy transfer

$$E = \hbar\omega = E_i - E_f$$



Changes also the energy transfer

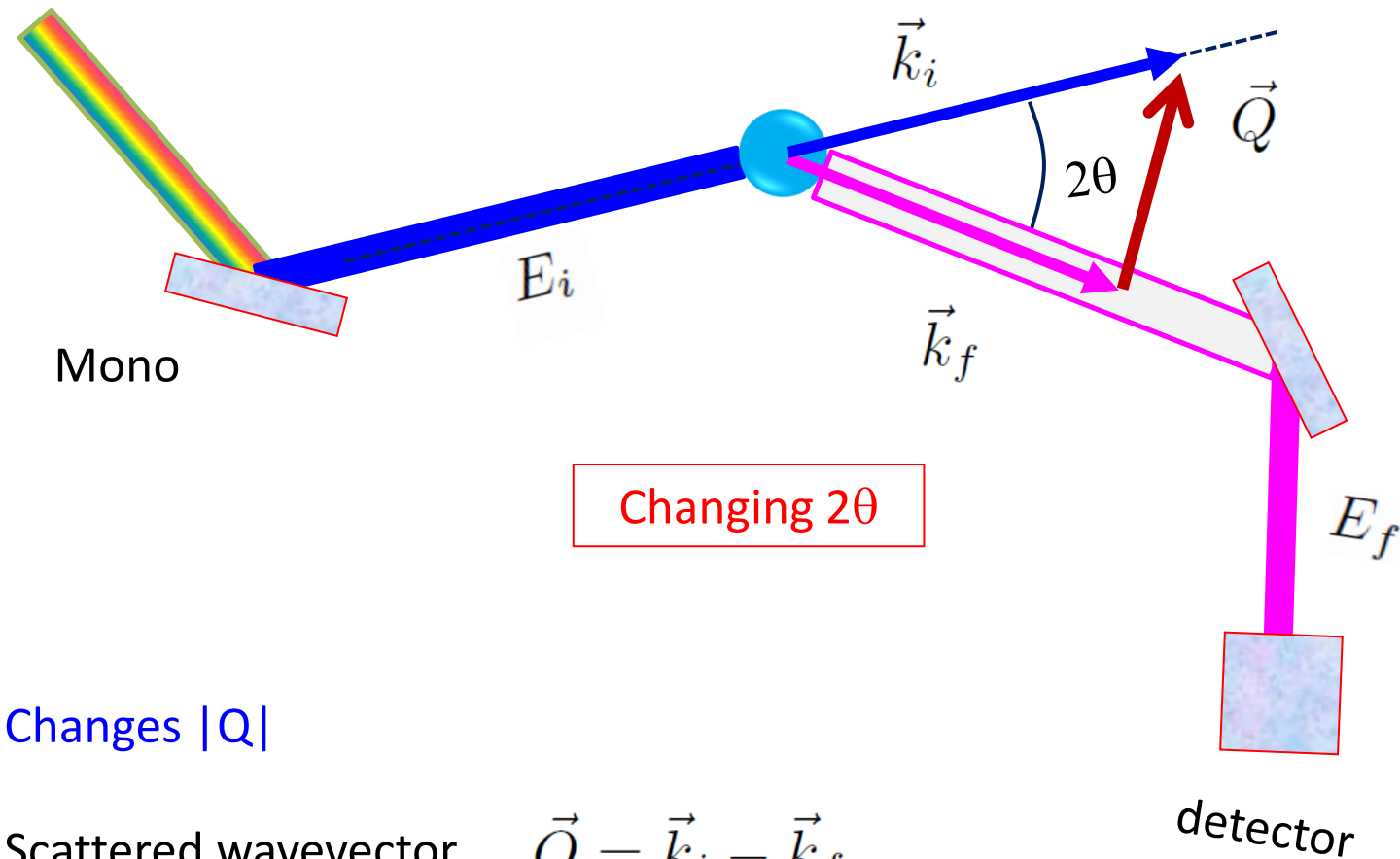
$$E = \hbar\omega = E_i - E_f$$



Changes $|Q|$

Scattered wavevector $\vec{Q} = \vec{k}_i - \vec{k}_f$

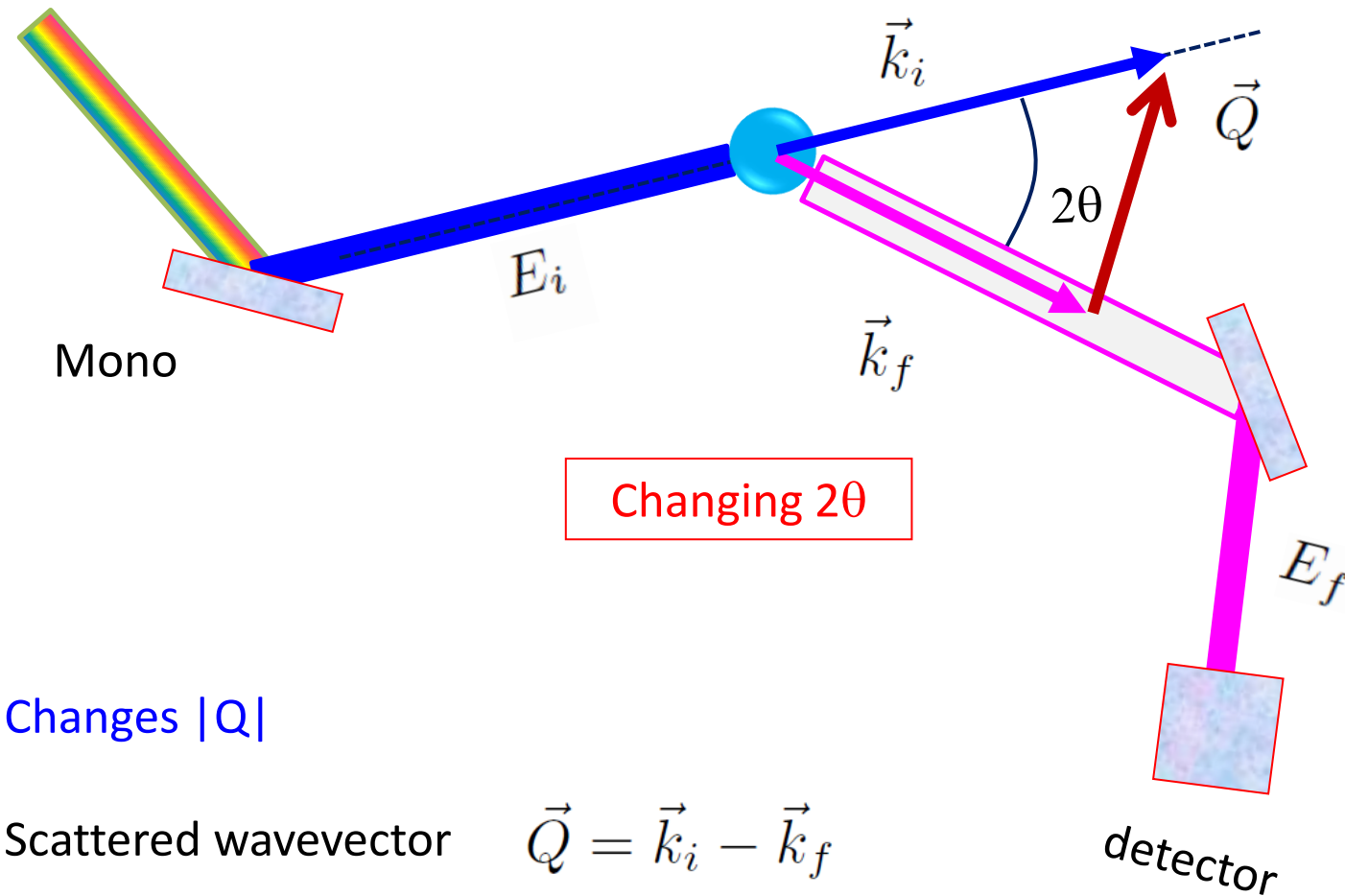
$$Q^2 = k_i^2 + k_f^2 - 2k_i k_f \cos 2\theta$$



Changes $|Q|$

Scattered wavevector $\vec{Q} = \vec{k}_i - \vec{k}_f$

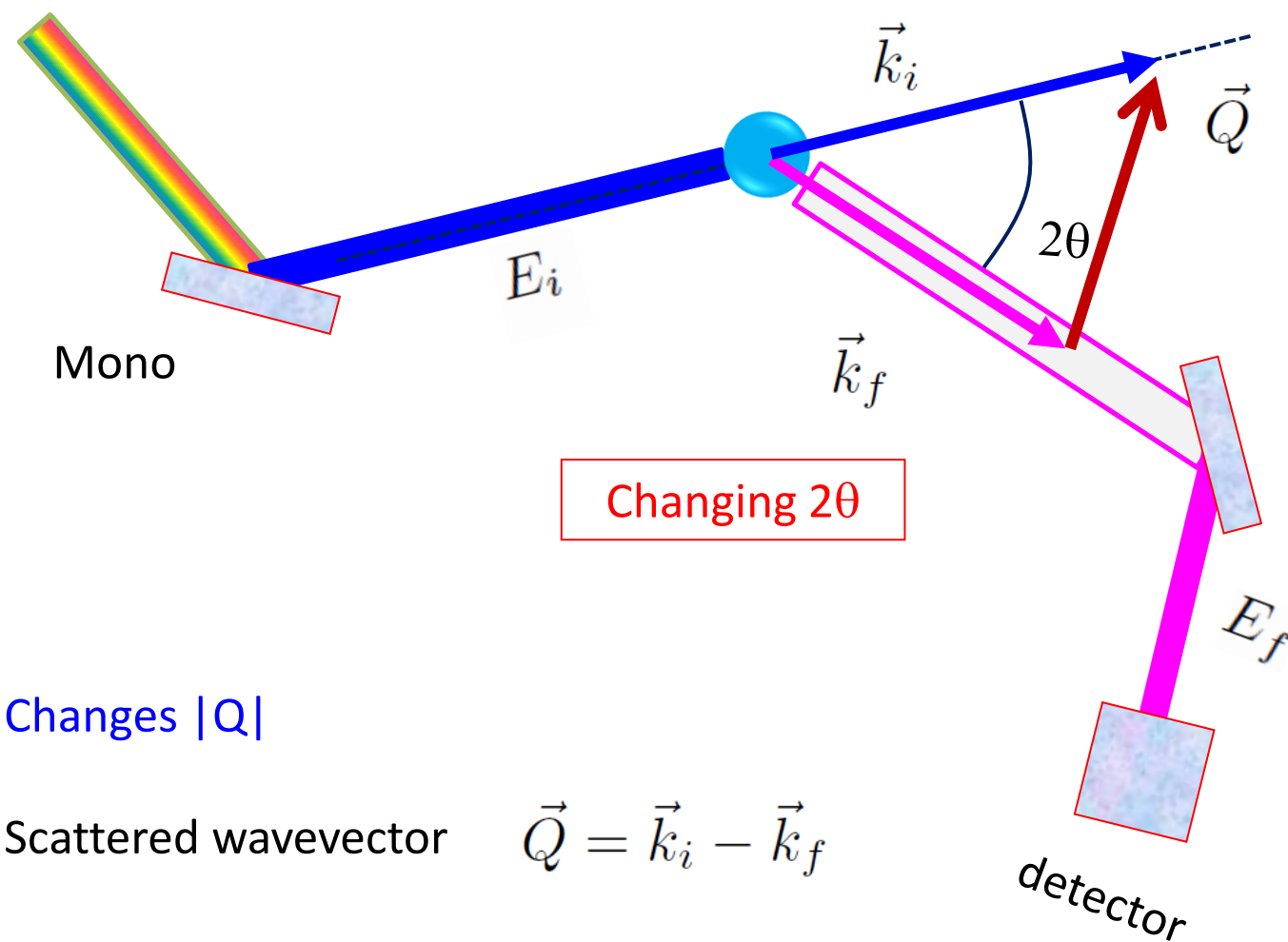
$$Q^2 = k_i^2 + k_f^2 - 2k_i k_f \cos 2\theta$$



Changes $|Q|$

Scattered wavevector $\vec{Q} = \vec{k}_i - \vec{k}_f$

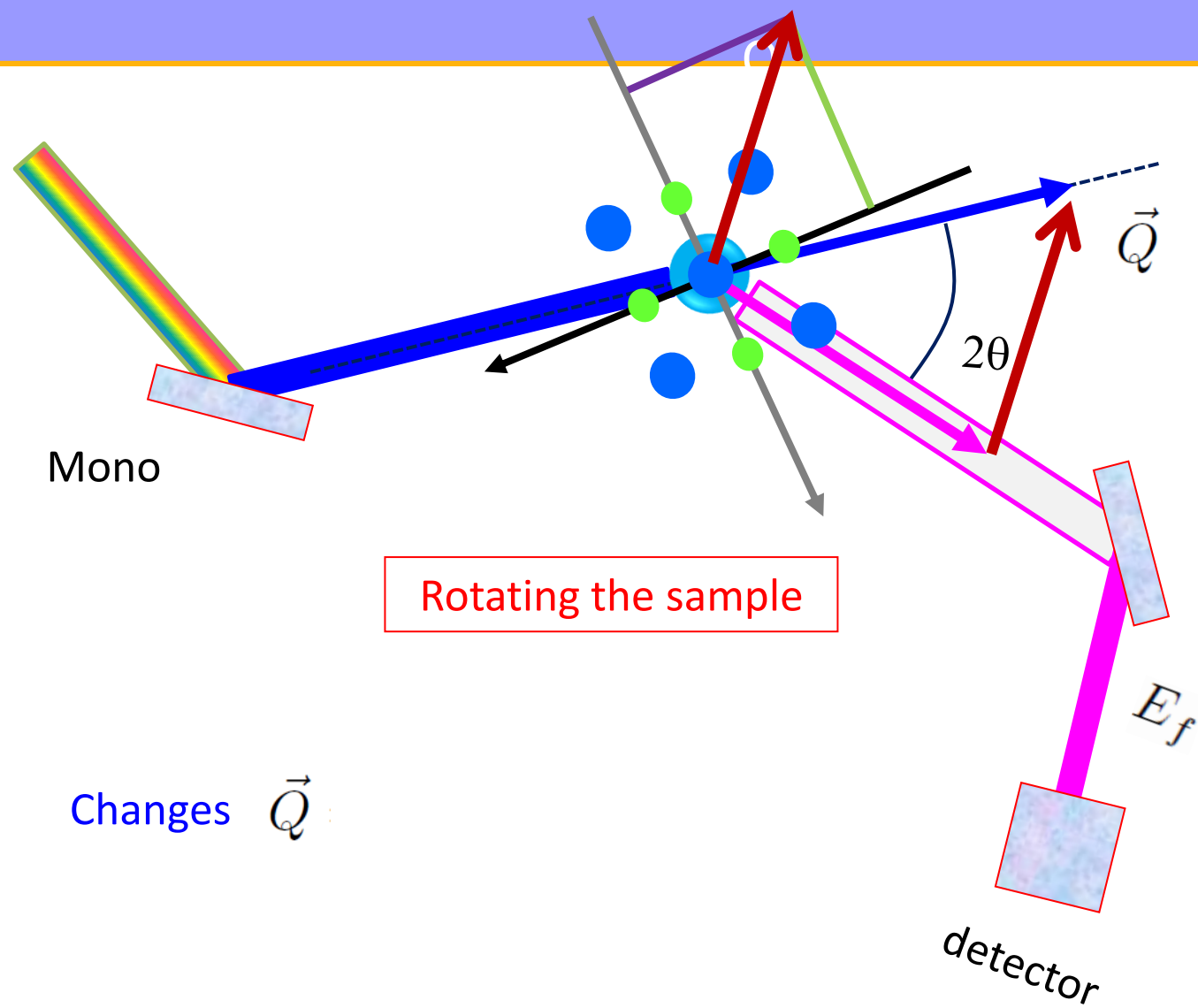
$$Q^2 = k_i^2 + k_f^2 - 2k_i k_f \cos 2\theta$$

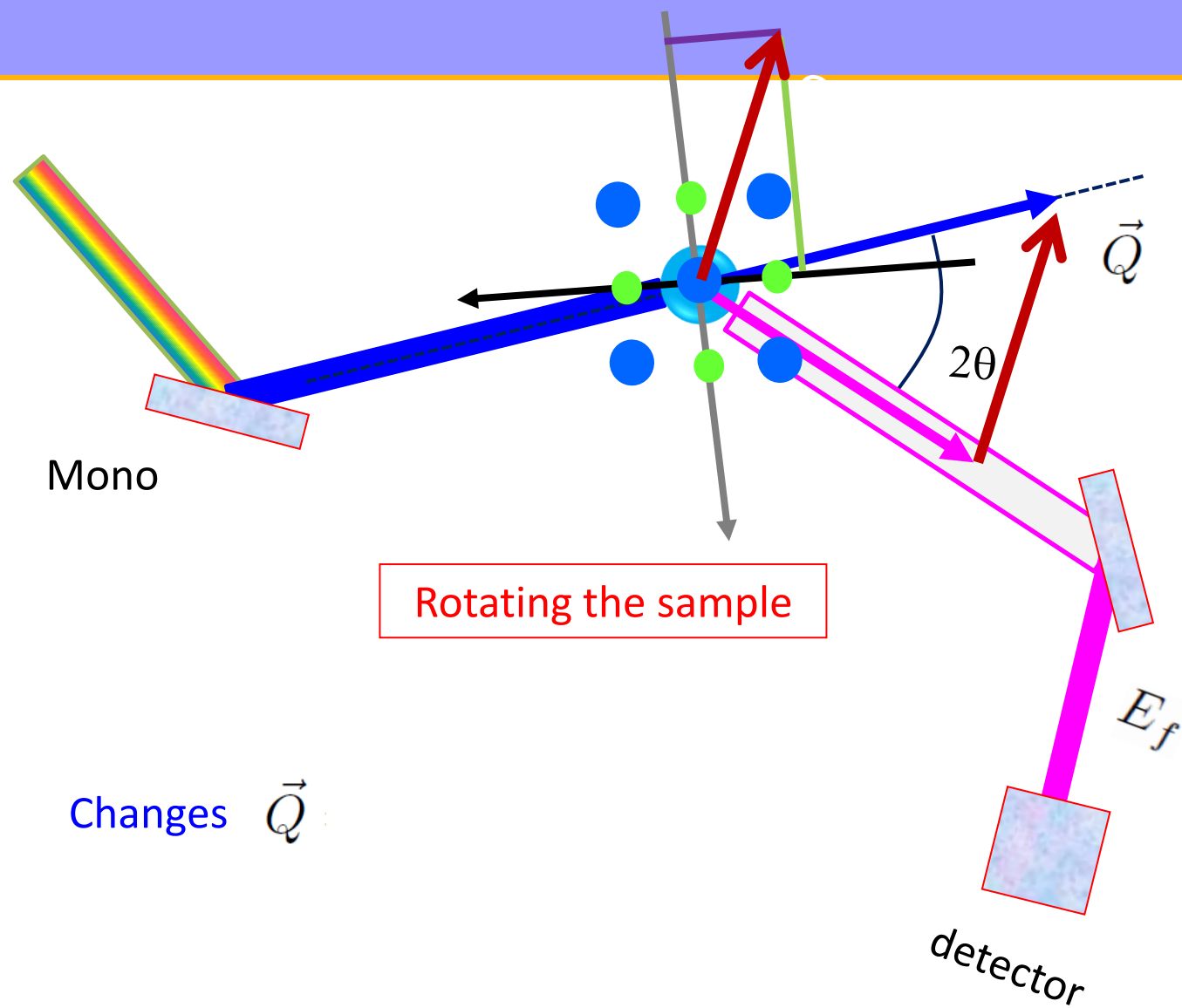


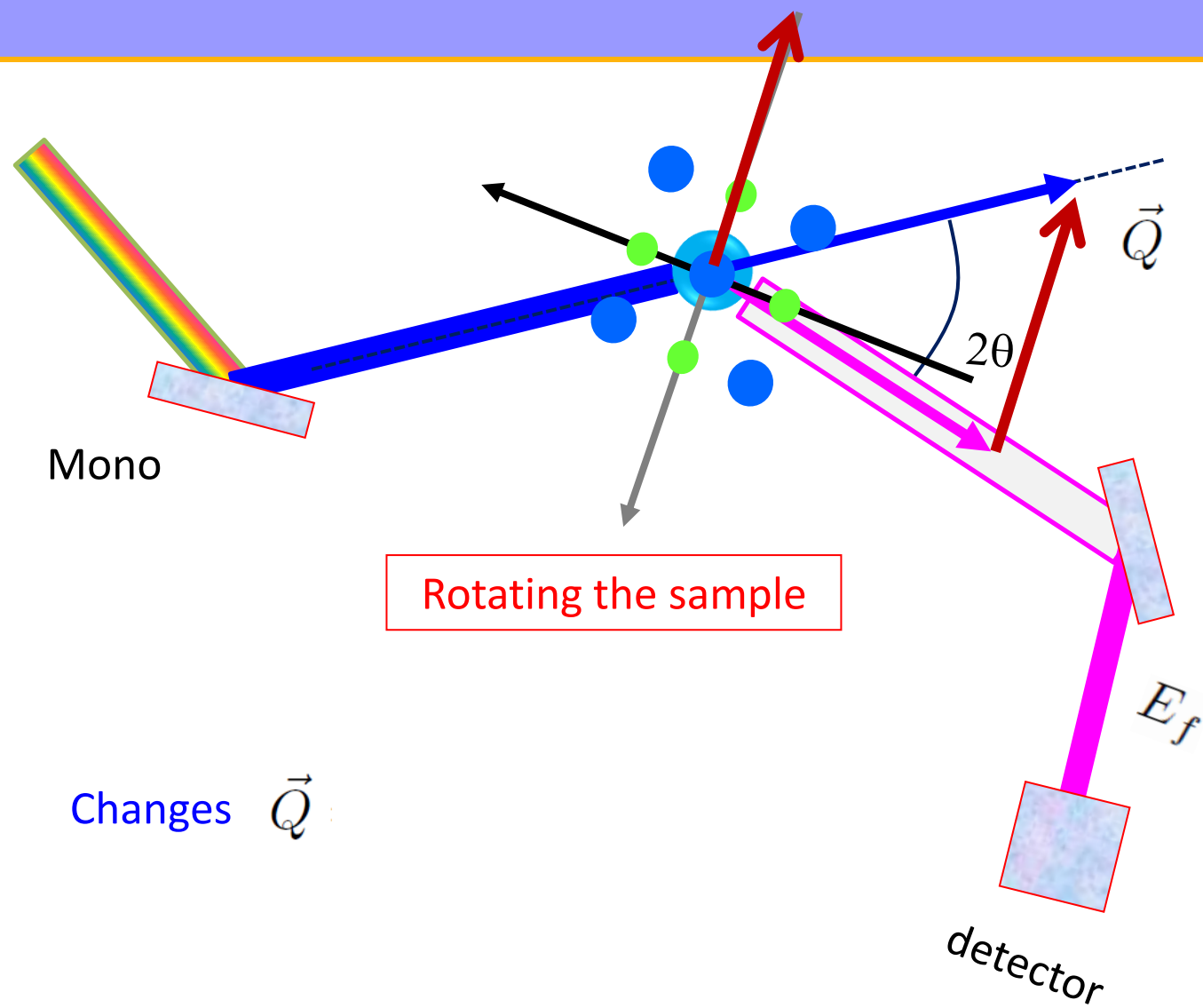
Changes $|Q|$

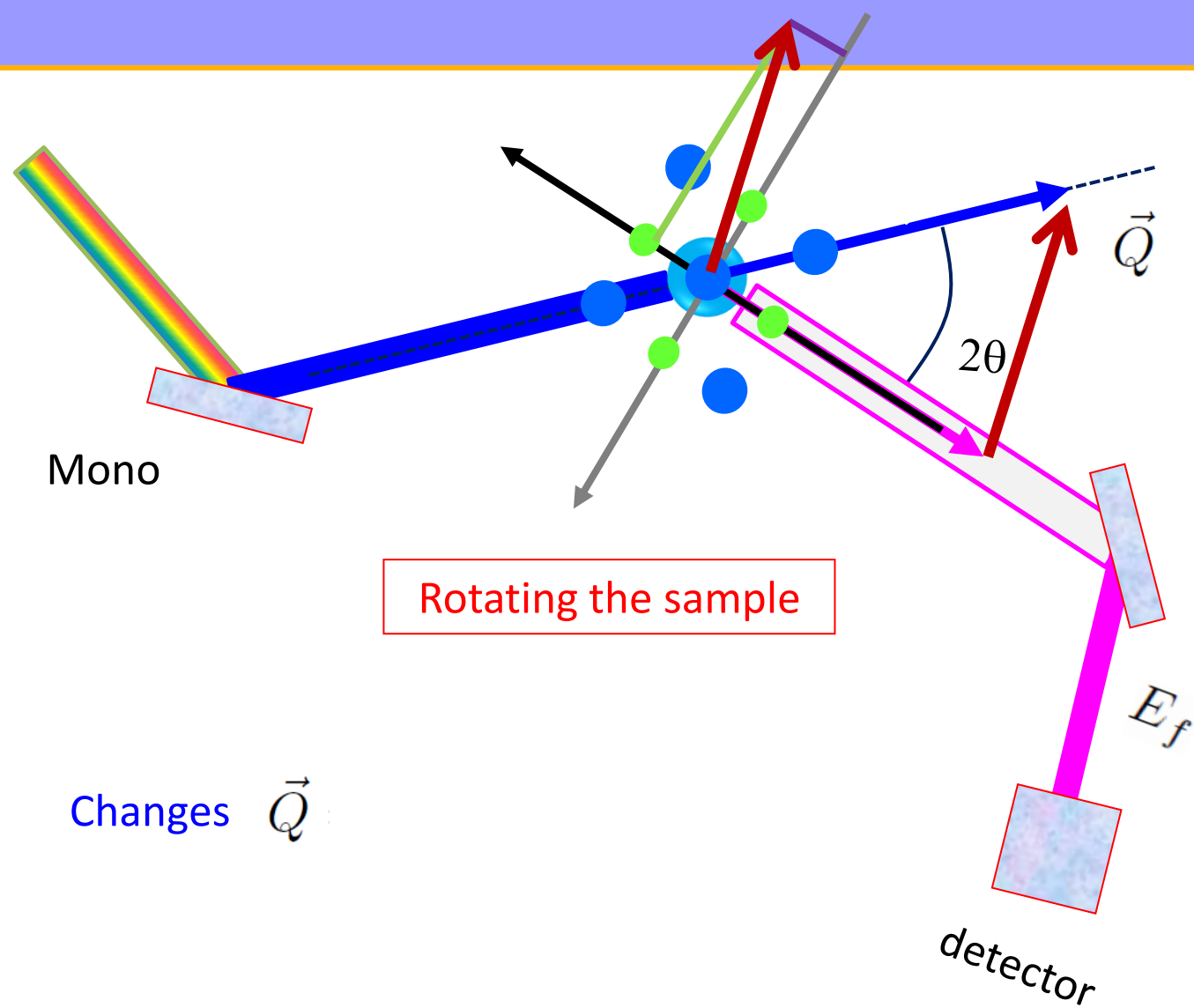
Scattered wavevector $\vec{Q} = \vec{k}_i - \vec{k}_f$

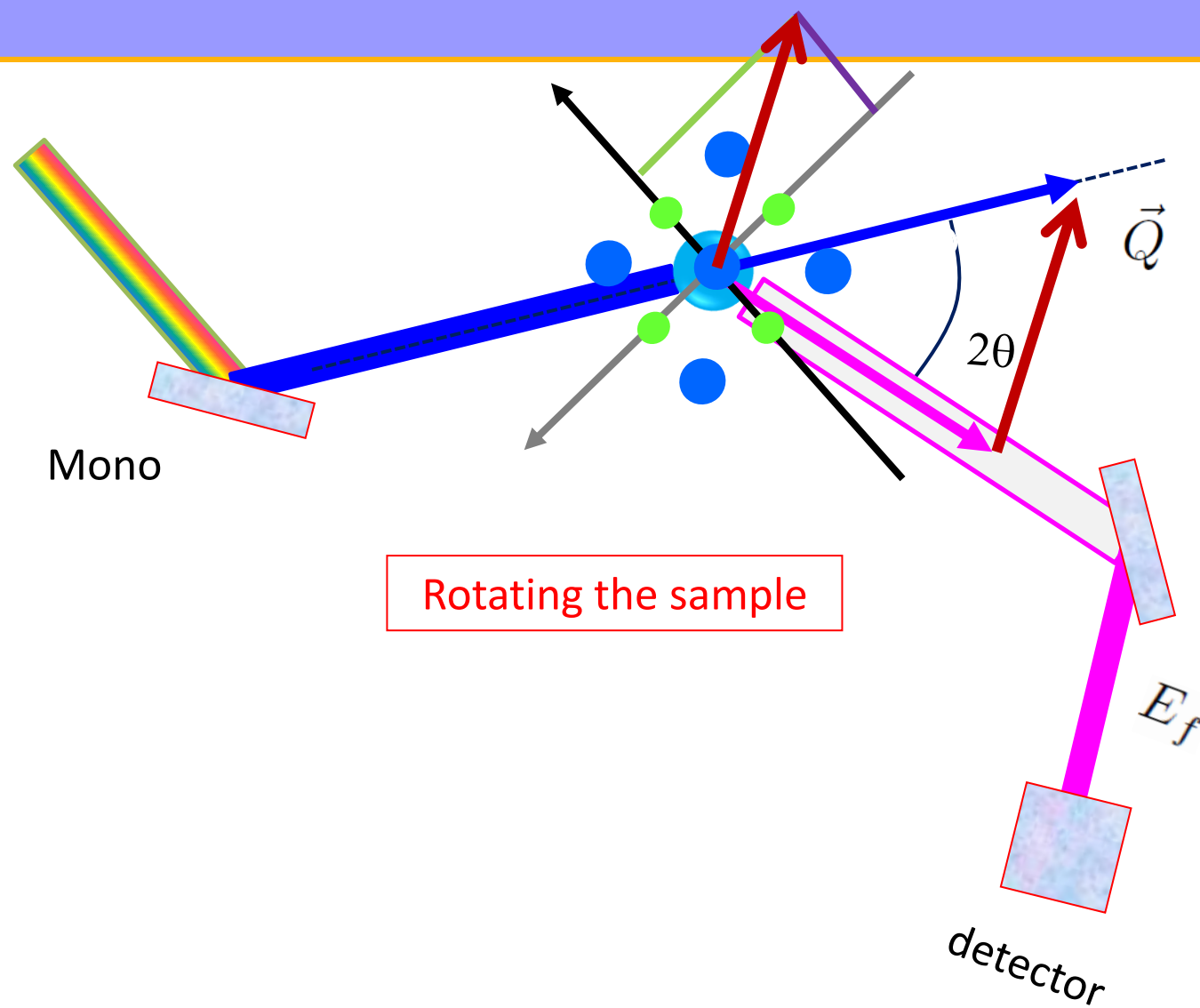
$$Q^2 = k_i^2 + k_f^2 - 2k_i k_f \cos 2\theta$$







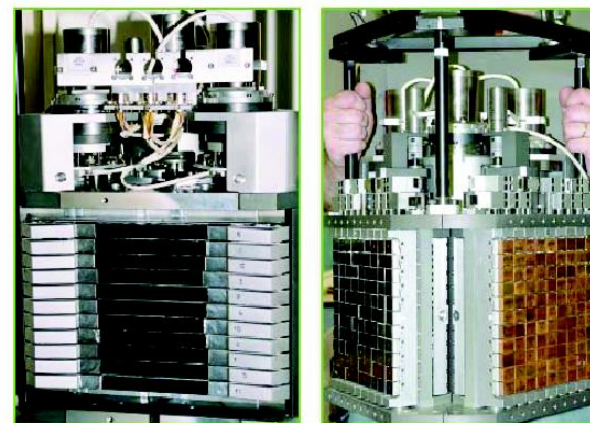
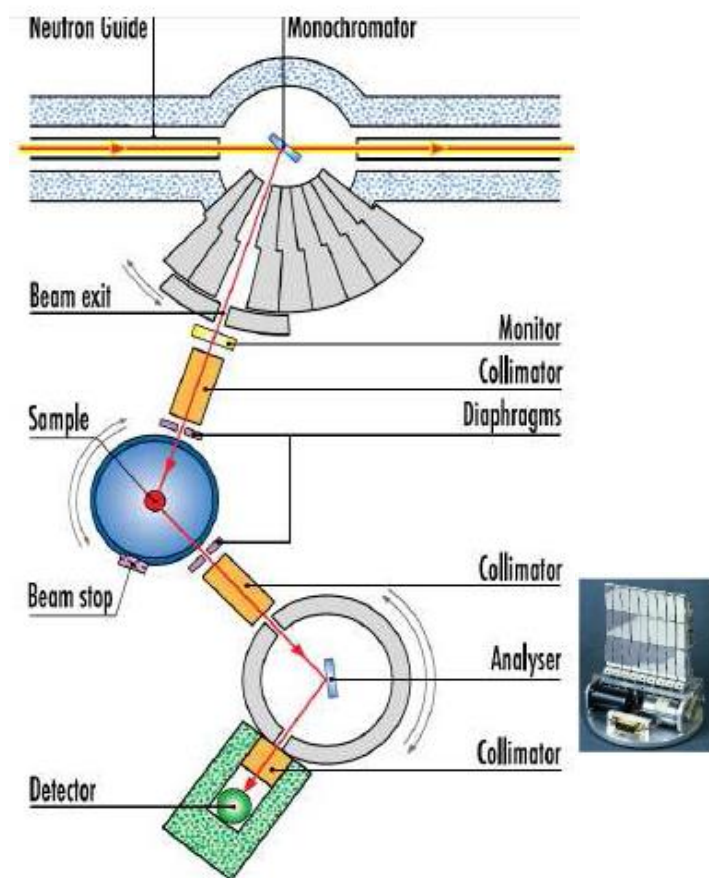


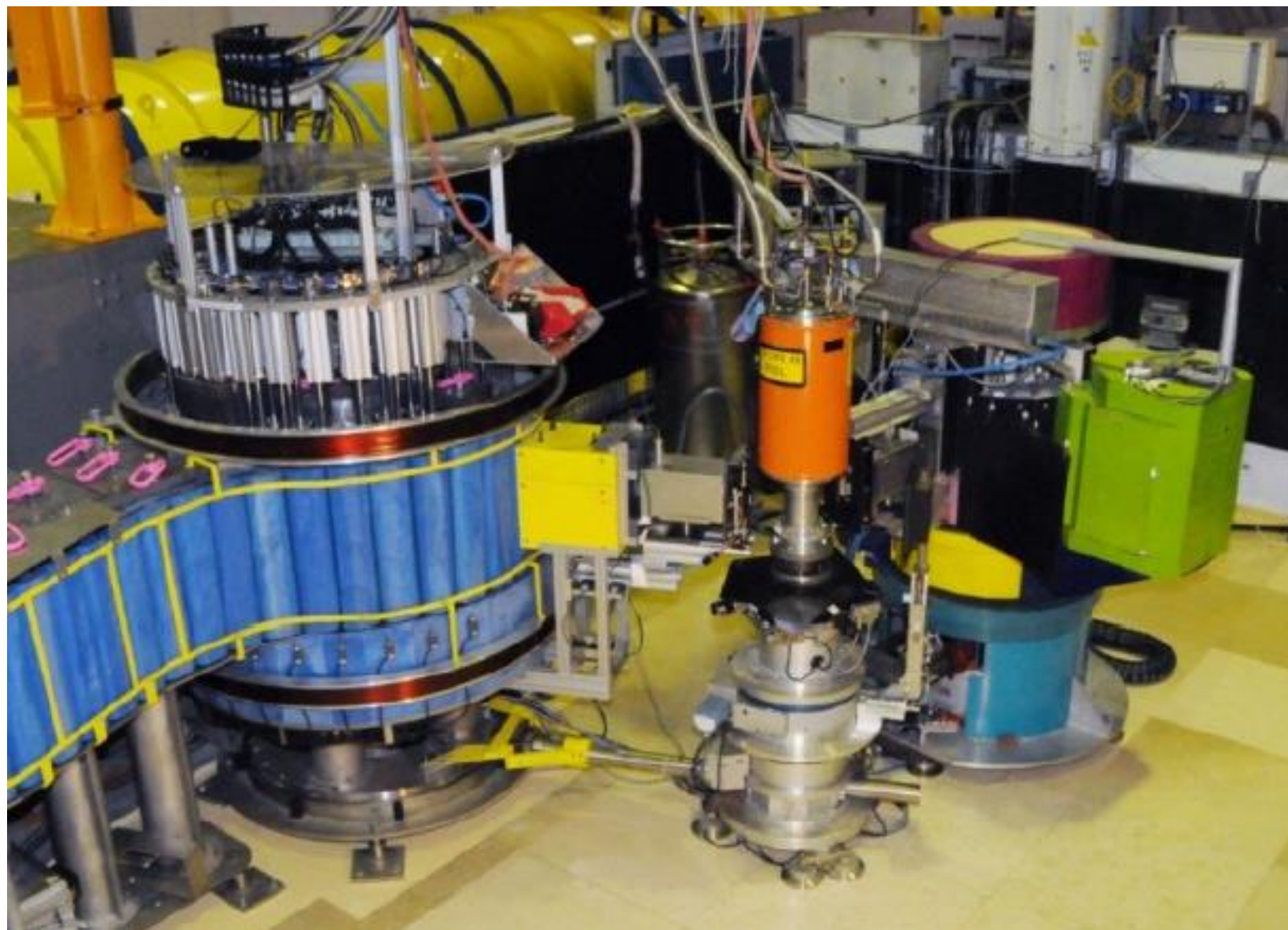


Time consuming



Triple axes spectrometer
(IN8, IN20, IN12, IN22 @ ILL)





6. What about the neutron spin ?
Is it possible to manipulate it?



Polarized neutrons

$$I_x^{++} \propto |N|^2$$

$$I_x^{--} \propto |N|^2$$

$$I_x^{+-} \propto |M_{\perp}|^2 - M_{ch}$$

$$I_x^{-+} \propto |M_{\perp}|^2 + M_{ch}$$

$$I_y^{++} \propto |N|^2 + |M_{\perp}^y|^2 + R_y$$

$$I_y^{--} \propto |N|^2 + |M_{\perp}^y|^2 - R_y$$

$$I_y^{+-} \propto |M_{\perp}^z|^2$$

$$I_y^{-+} \propto |M_{\perp}^z|^2$$

$$I_z^{++} \propto |N|^2 + |M_{\perp}^z|^2 + R_z$$

$$I_z^{--} \propto |N|^2 + |M_{\perp}^z|^2 - R_z$$

$$I_z^{+-} \propto |M_{\perp}^y|^2$$

$$I_z^{-+} \propto |M_{\perp}^y|^2$$

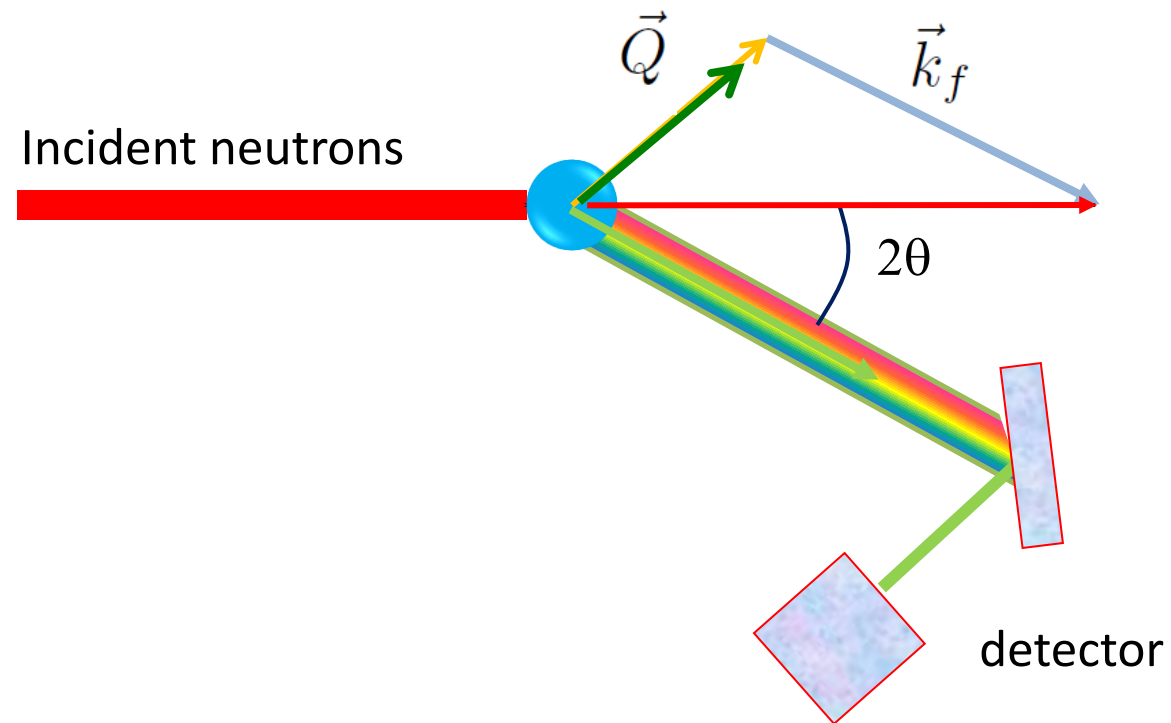
Depending on the spin direction wrt Q, new correlation functions appear

$$\mathbf{M} = \mathbf{M}^z + \mathbf{M}^y$$

and

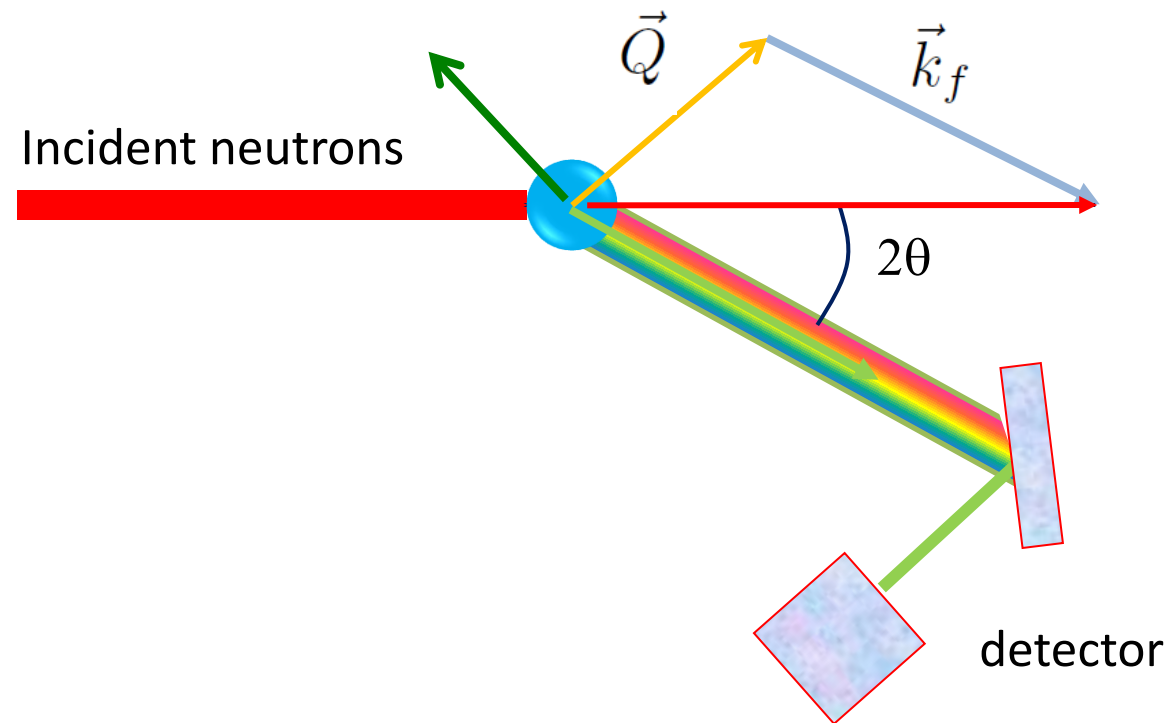
$$\mathbf{M}^y \quad \mathbf{M}^z \quad \mathbf{M}_{ch}$$

Polarized neutrons



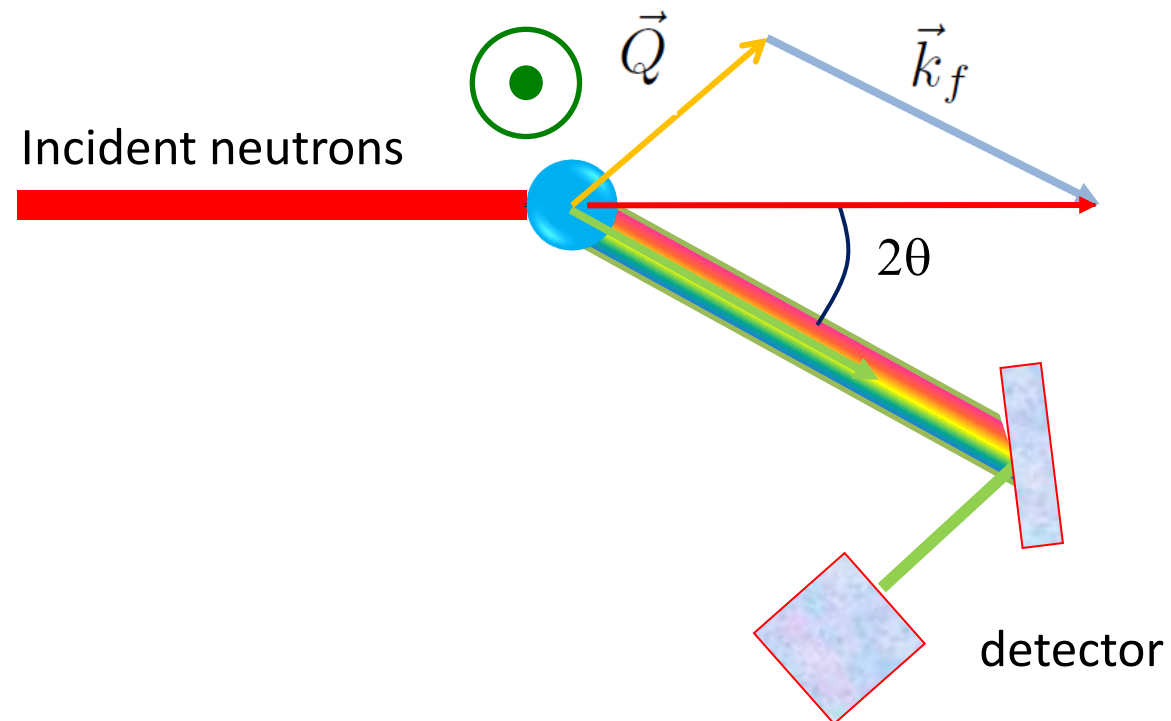
Neutron spin // $\vec{Q}$ (« x »)

Polarized neutrons



Neutron spin perpendicular to $\vec{Q}$ within the scattering plane (« y »)

Polarized neutrons



Neutron spin perpendicular to $\vec{Q}$ and to the scattering plane (« z »)

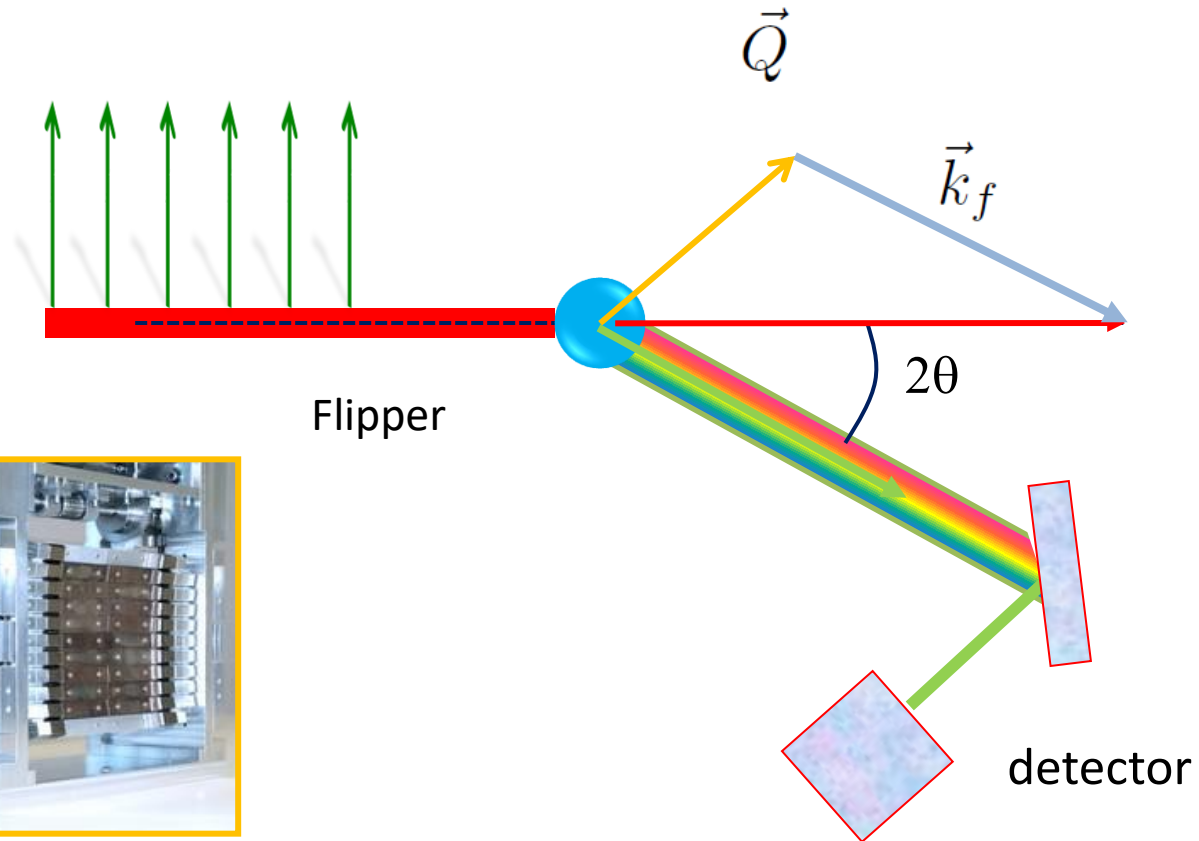
Incoming neutron spin



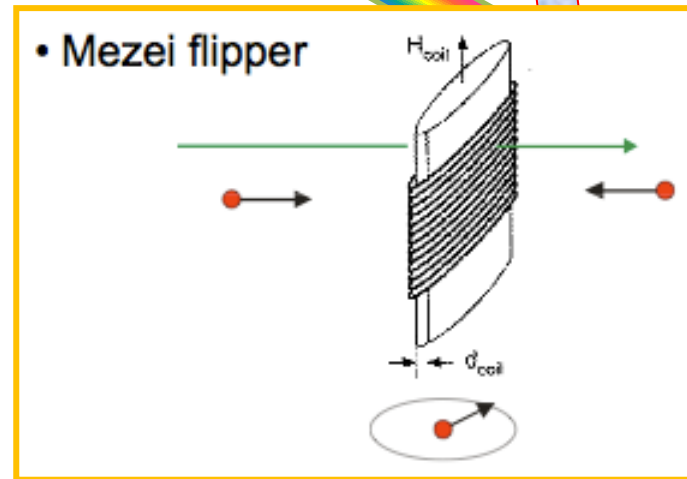
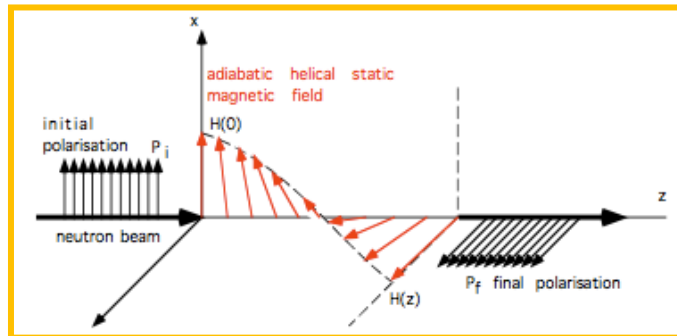
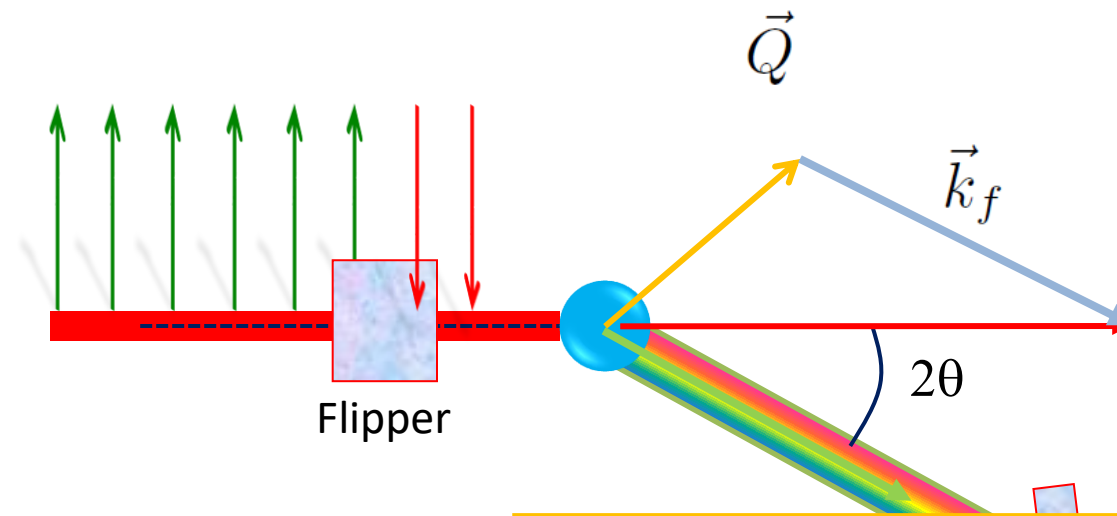
Bender



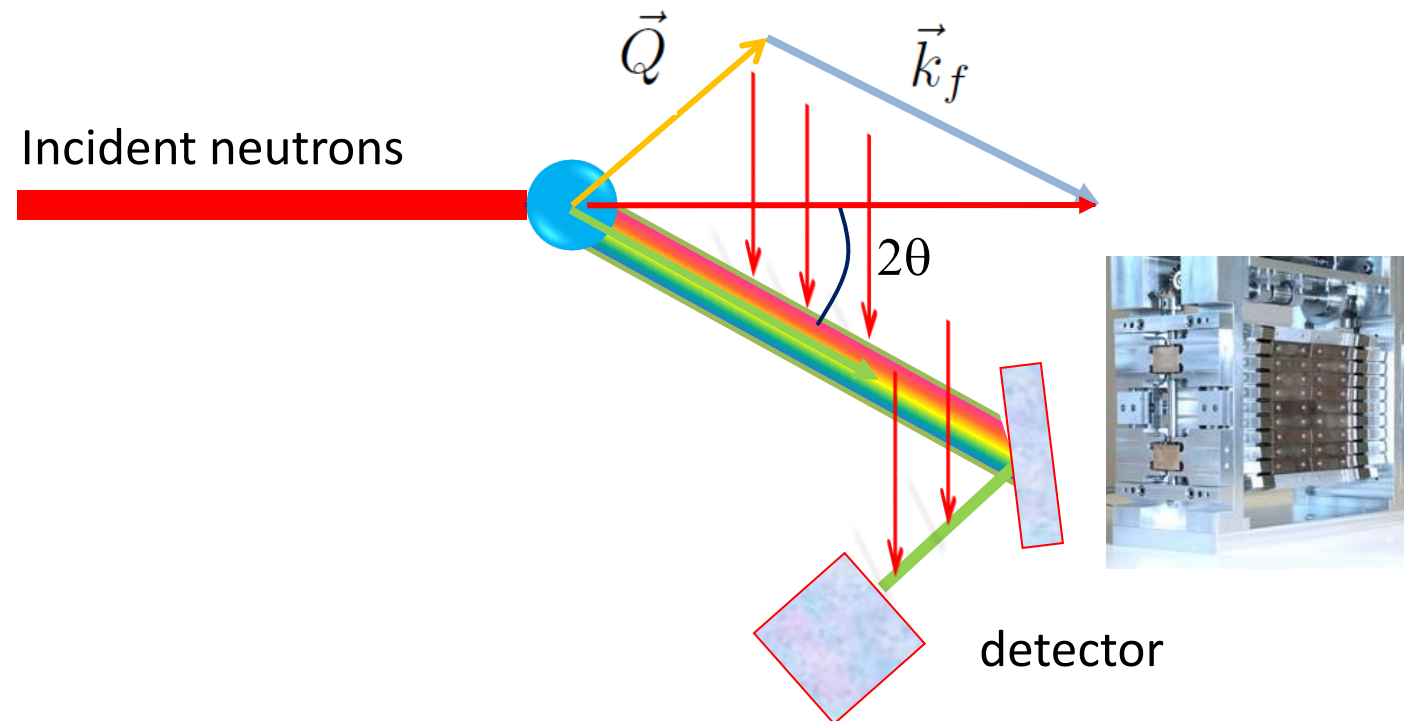
Mono
Heussler



Incoming neutron spin

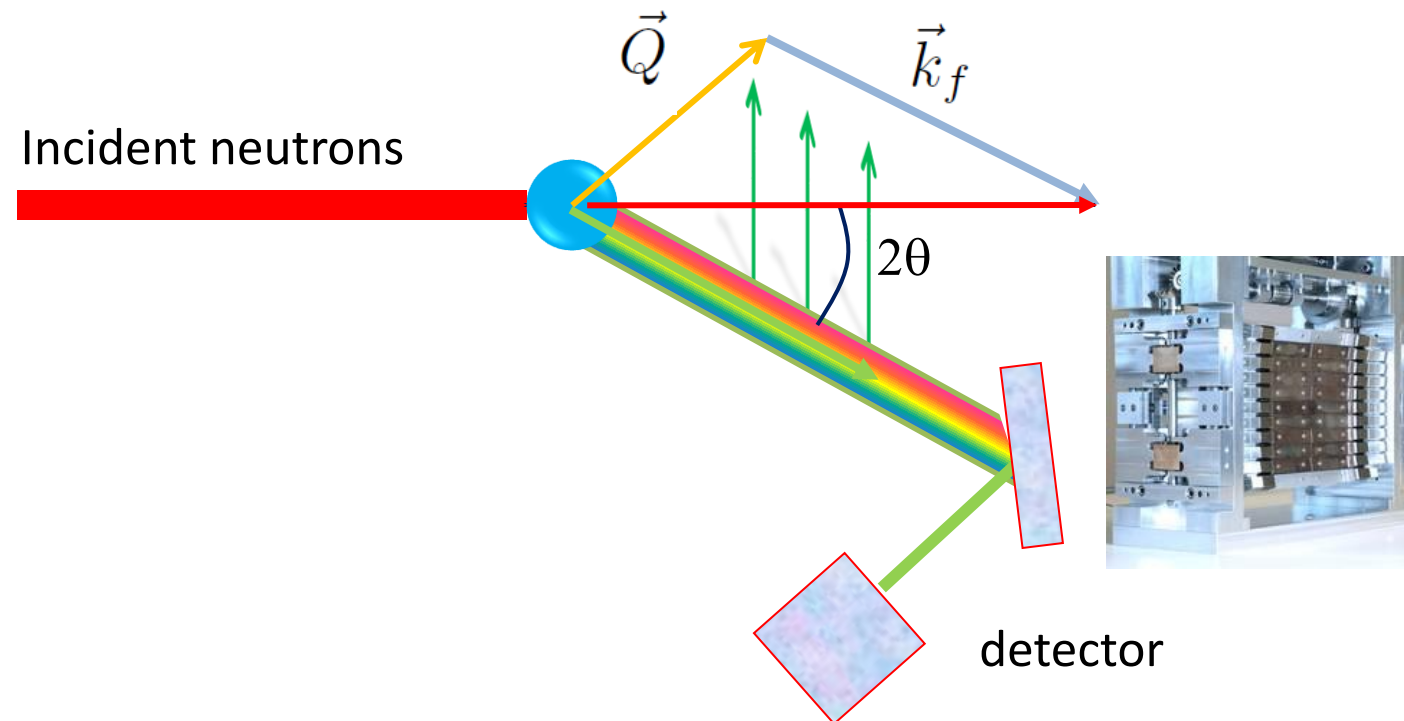


Scattered neutron spin



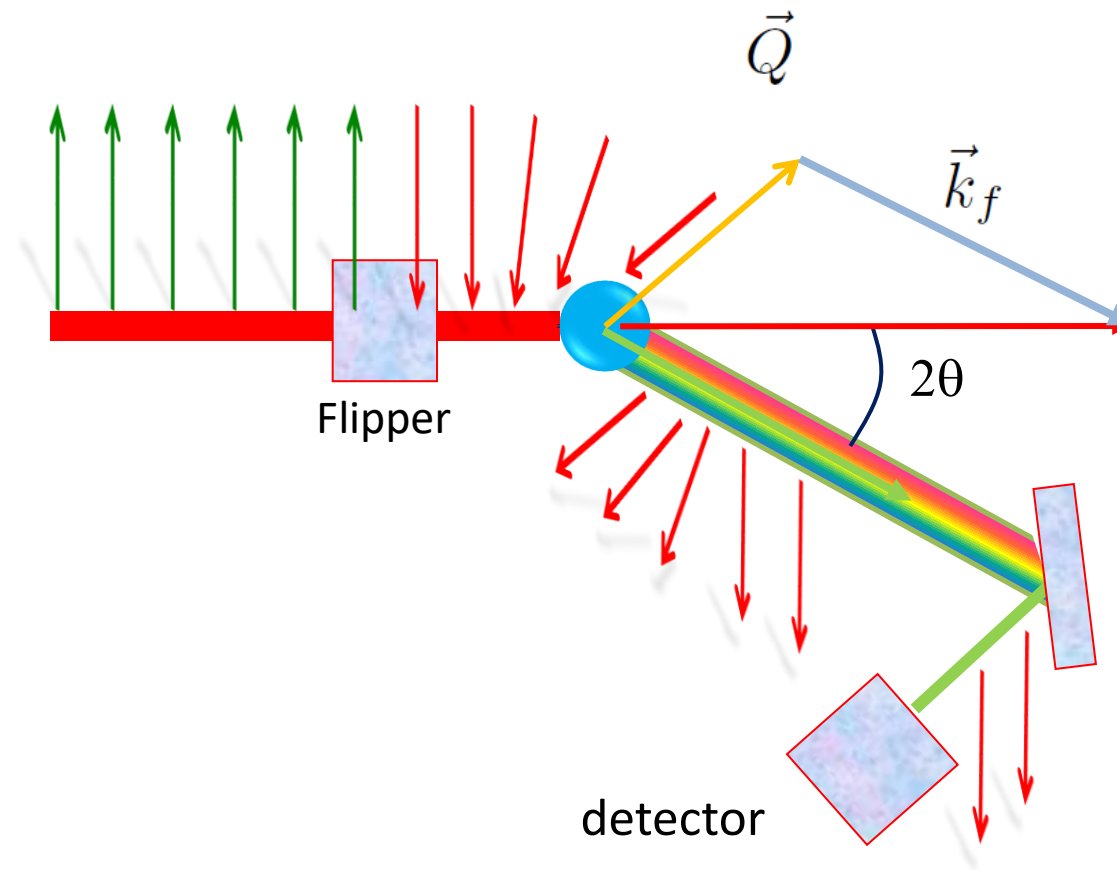
The analyzer selects the final polarization.
Projects onto a given direction (fixed by set up
of the instrument)

Scattered neutron spin



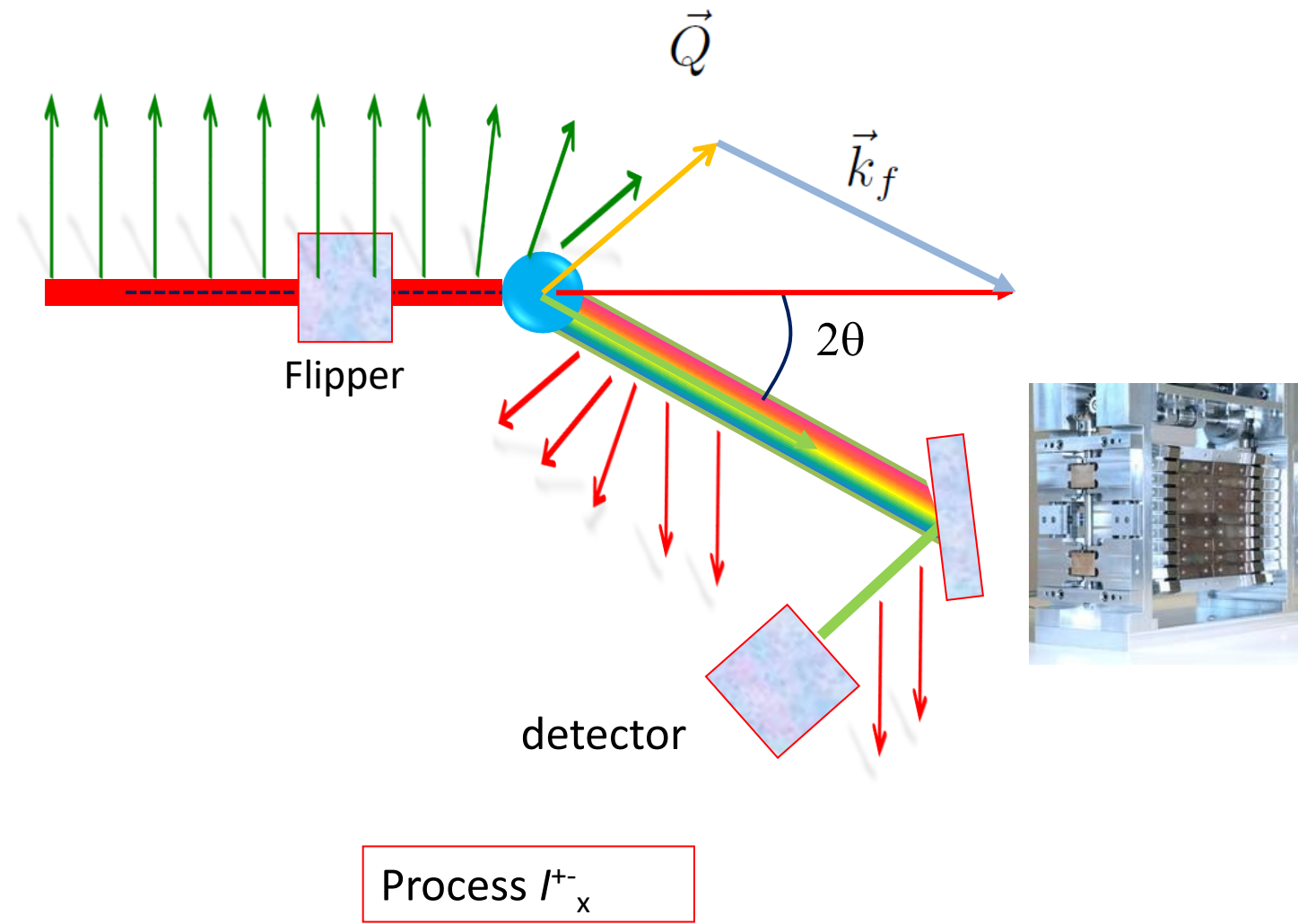
The analyzer selects the final polarization.
Projects onto a given direction (fixed by set up
of the instrument)

Polarized neutrons



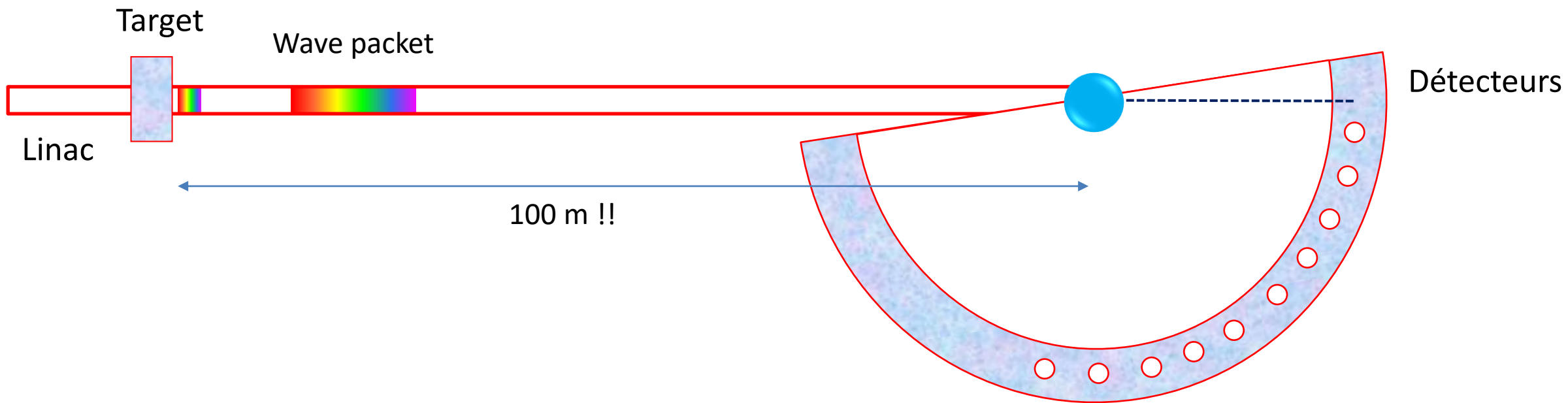
Process I^-_x

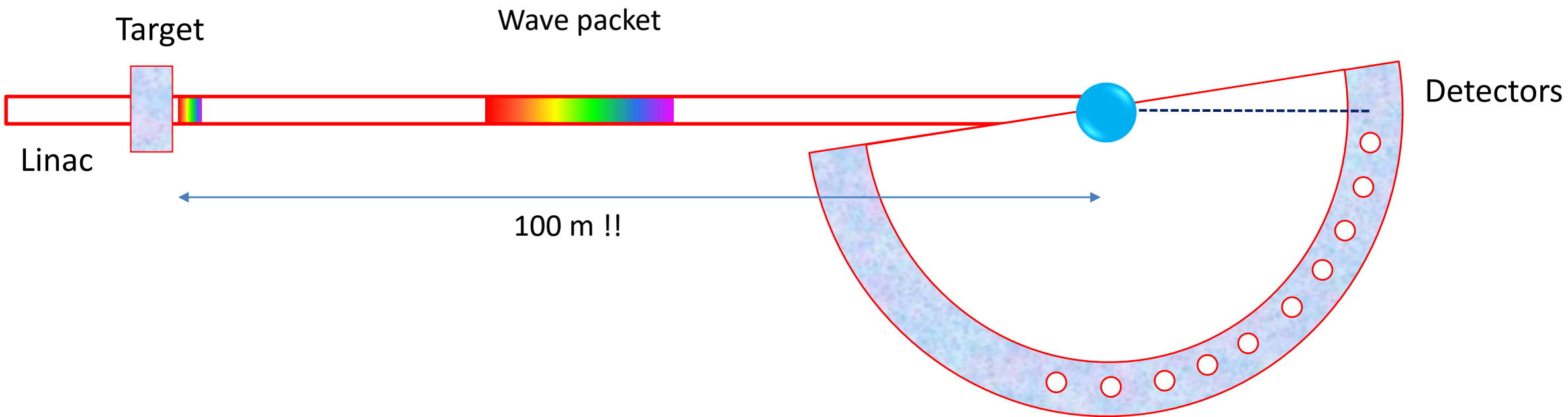
Polarized neutrons

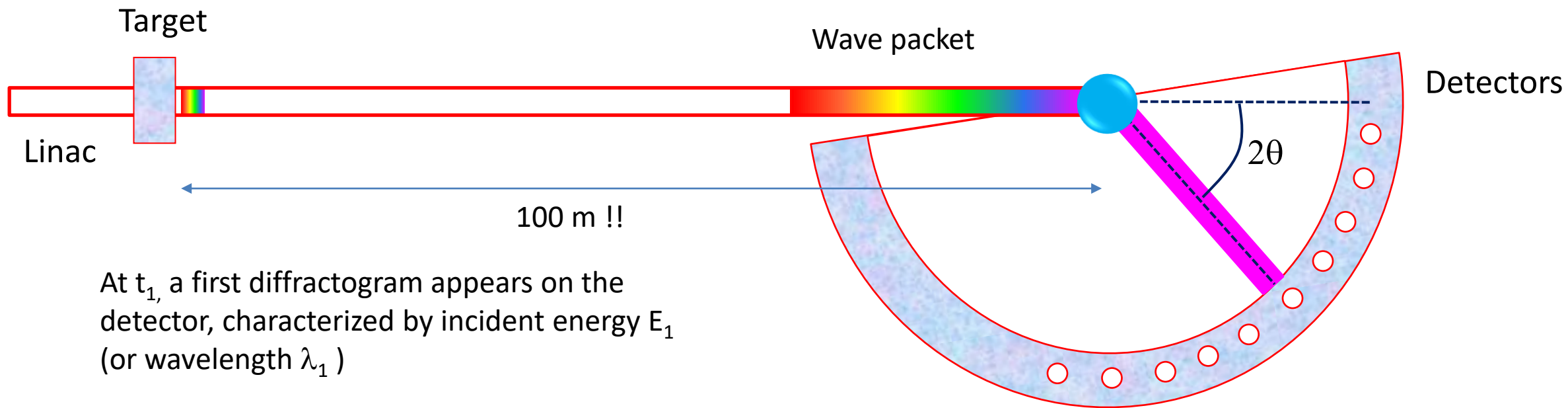


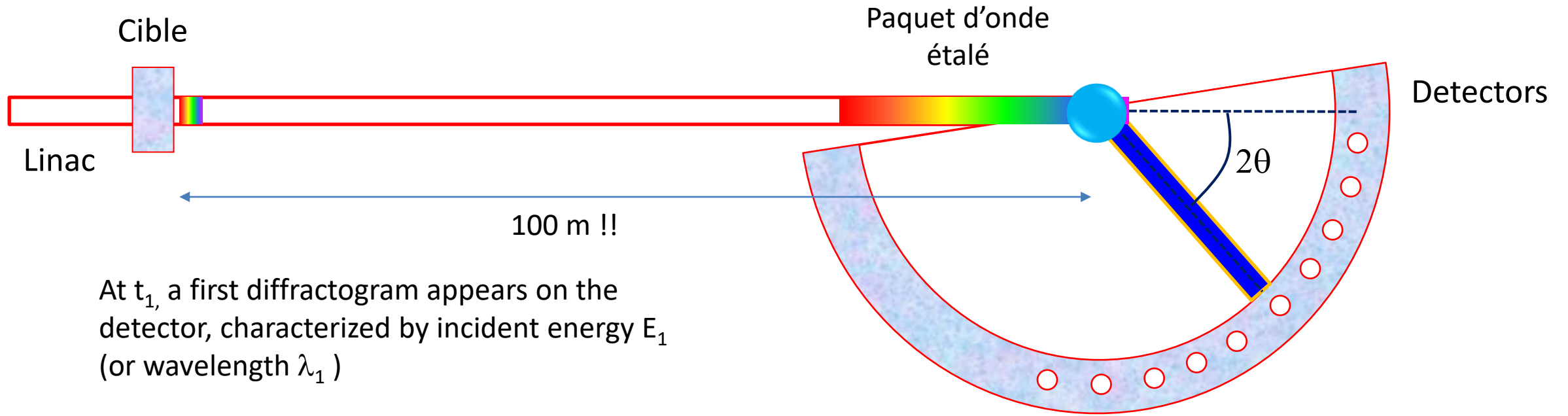
7. ESS and polychromatic pulses





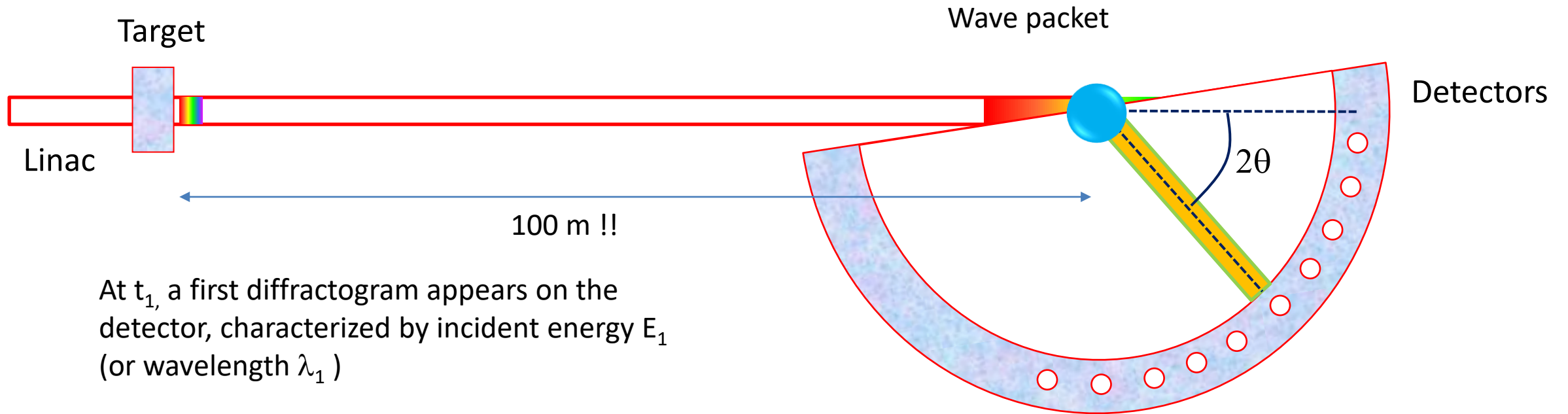






At t_1 , a first diffractogram appears on the detector, characterized by incident energy E_1 (or wavelength λ_1)

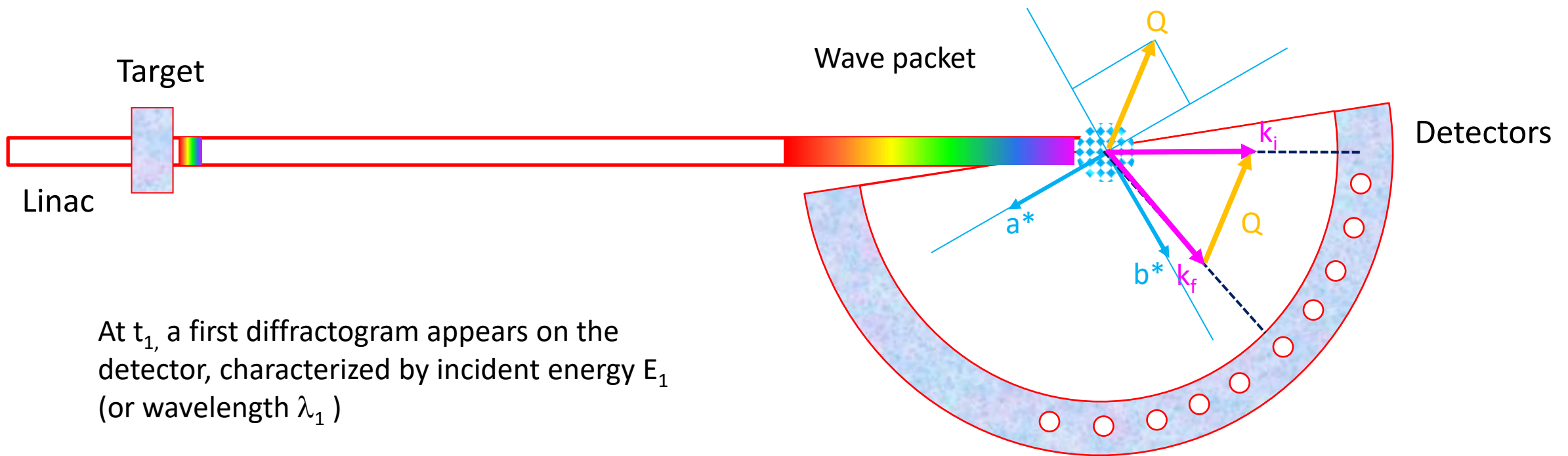
At $t_2 = t_1 + \delta t$, a new one forms, characterized by incident energy E_2 (wavelength λ_2)



At t_1 , a first diffractogram appears on the detector, characterized by incident energy E_1 (or wavelength λ_1)

At $t_2 = t_1 + \delta t$, a new one forms, characterized by incident energy E_2 (wavelength λ_2)

... and so on for E_i all in the wave packet.

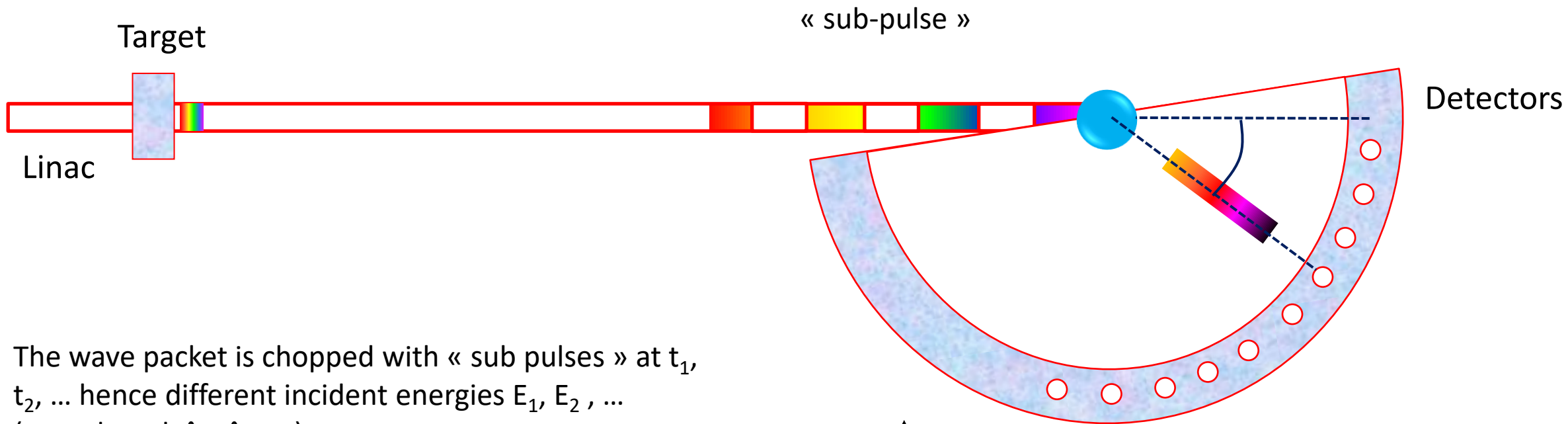


At t_1 , a first diffractogram appears on the detector, characterized by incident energy E_1 (or wavelength λ_1)

At $t_2 = t_1 + \delta t$, a new one forms, characterized by incident energy E_2 (wavelength λ_2)

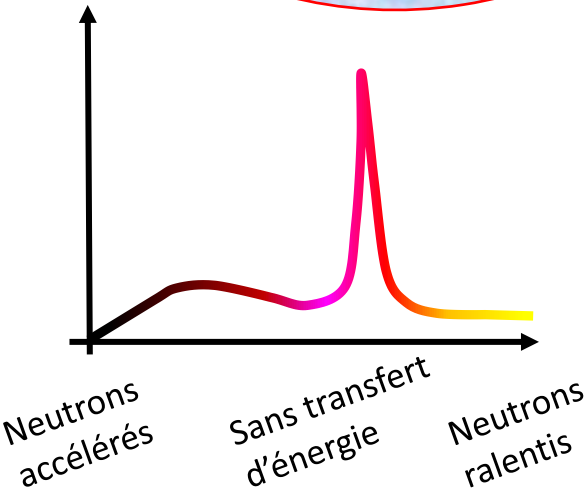
... and so on for E_i all in the wave packet.

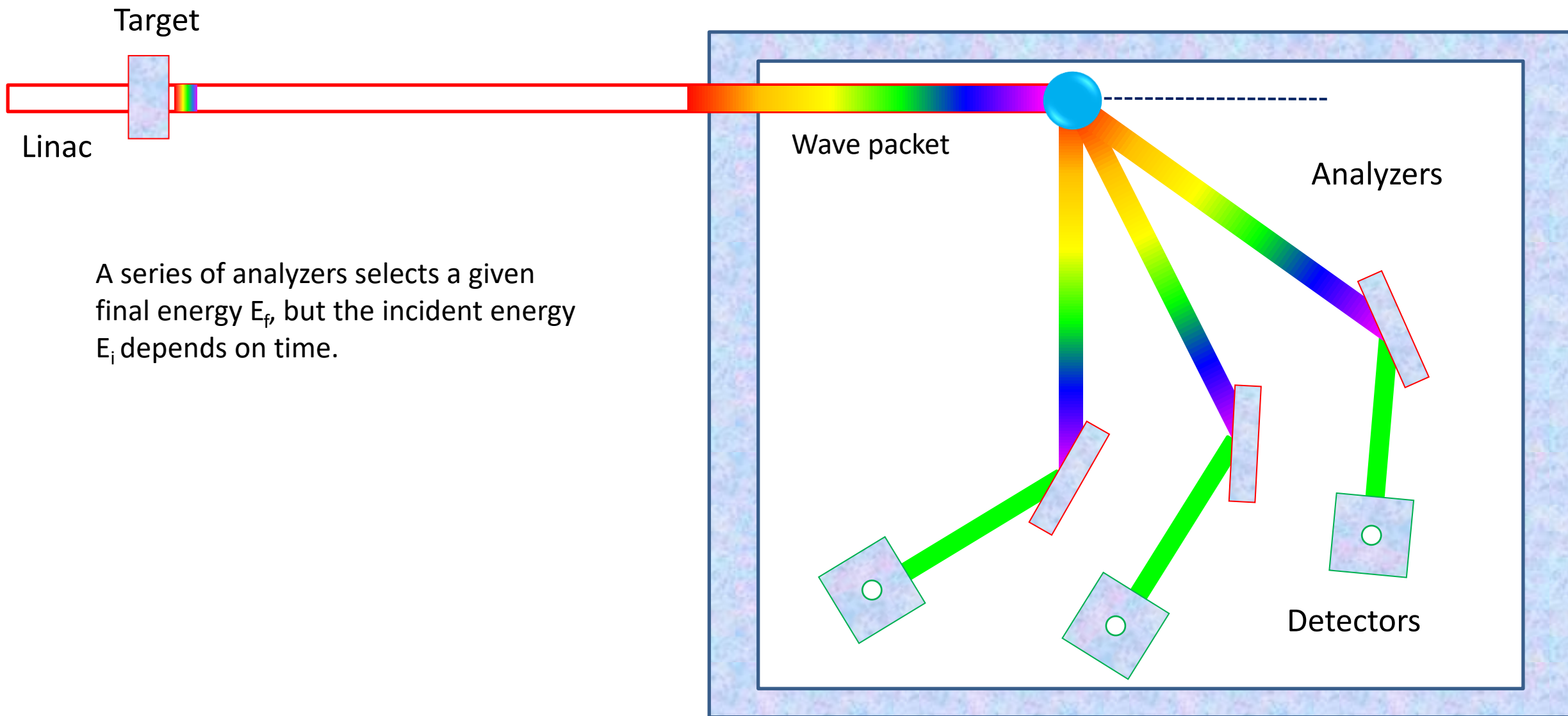
ISSUE = DATA REDUCTION

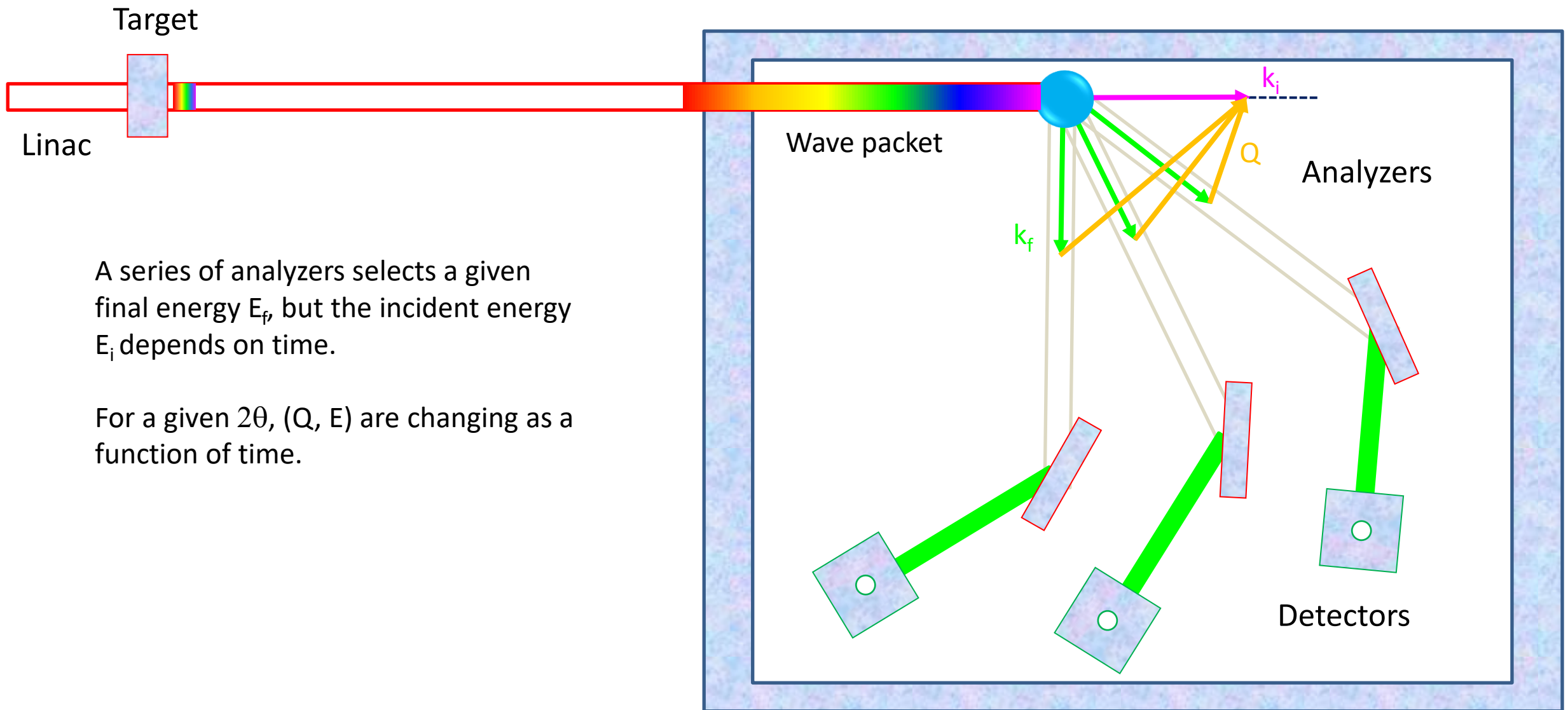


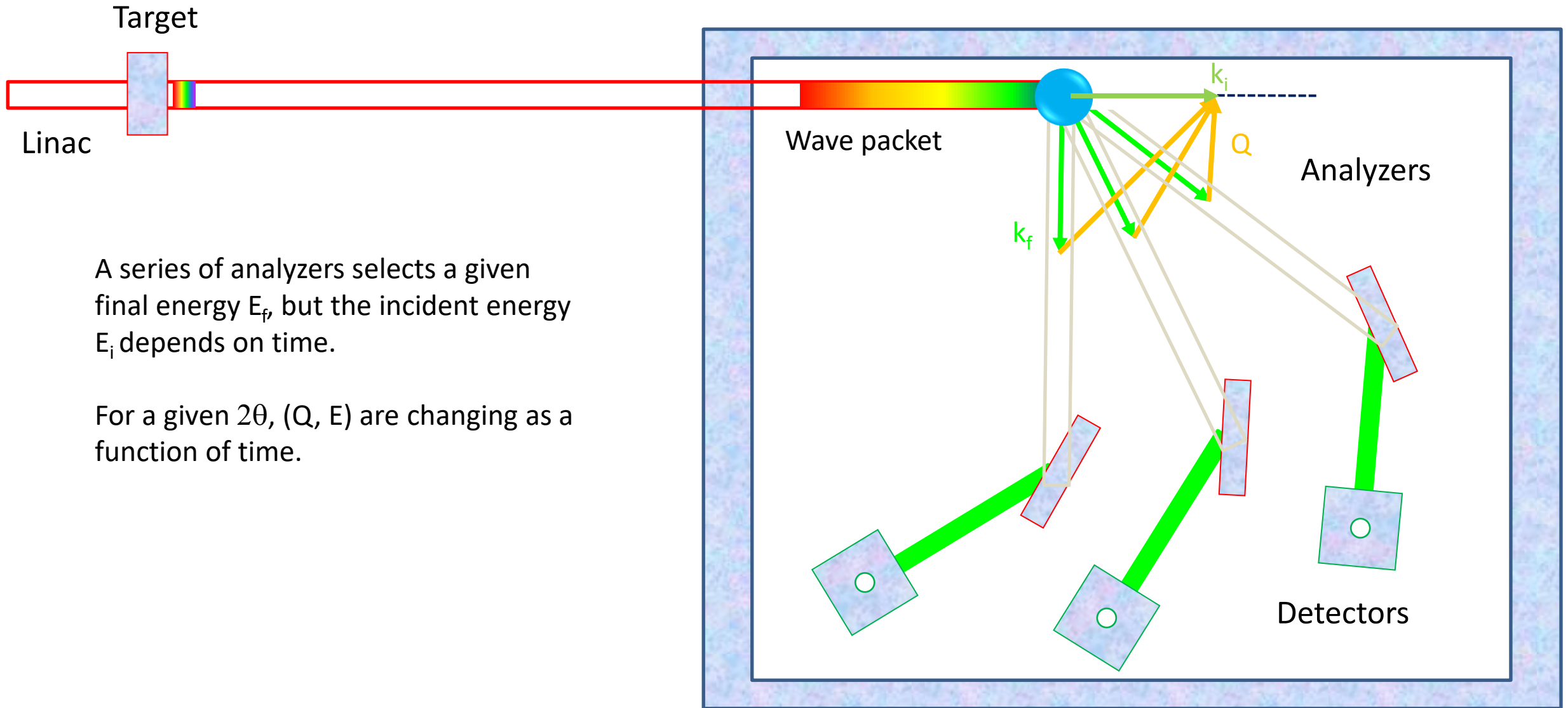
The wave packet is chopped with « sub pulses » at t_1 , t_2 , ... hence different incident energies E_1 , E_2 , ... (wave length λ_1 , λ_2 ...)

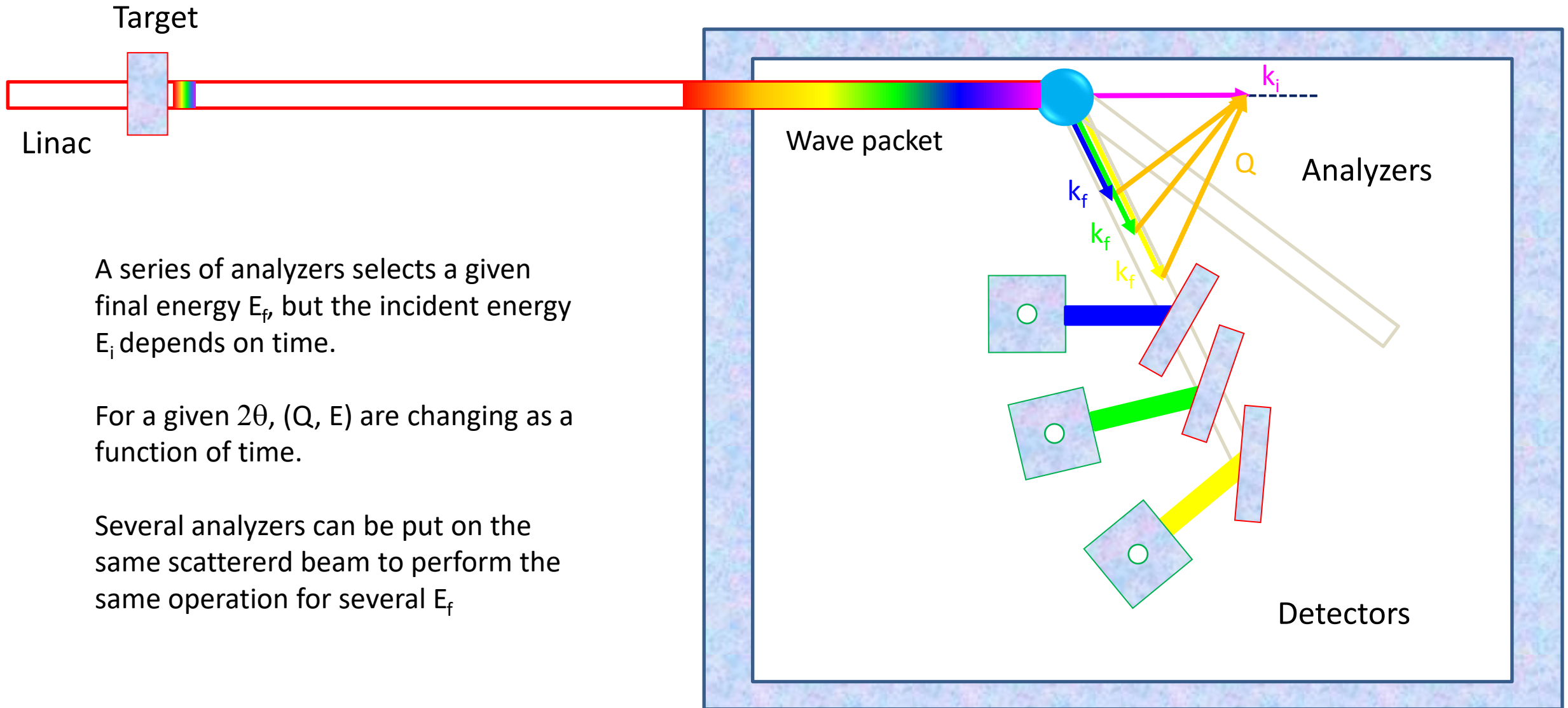
Similar to a standard TOF





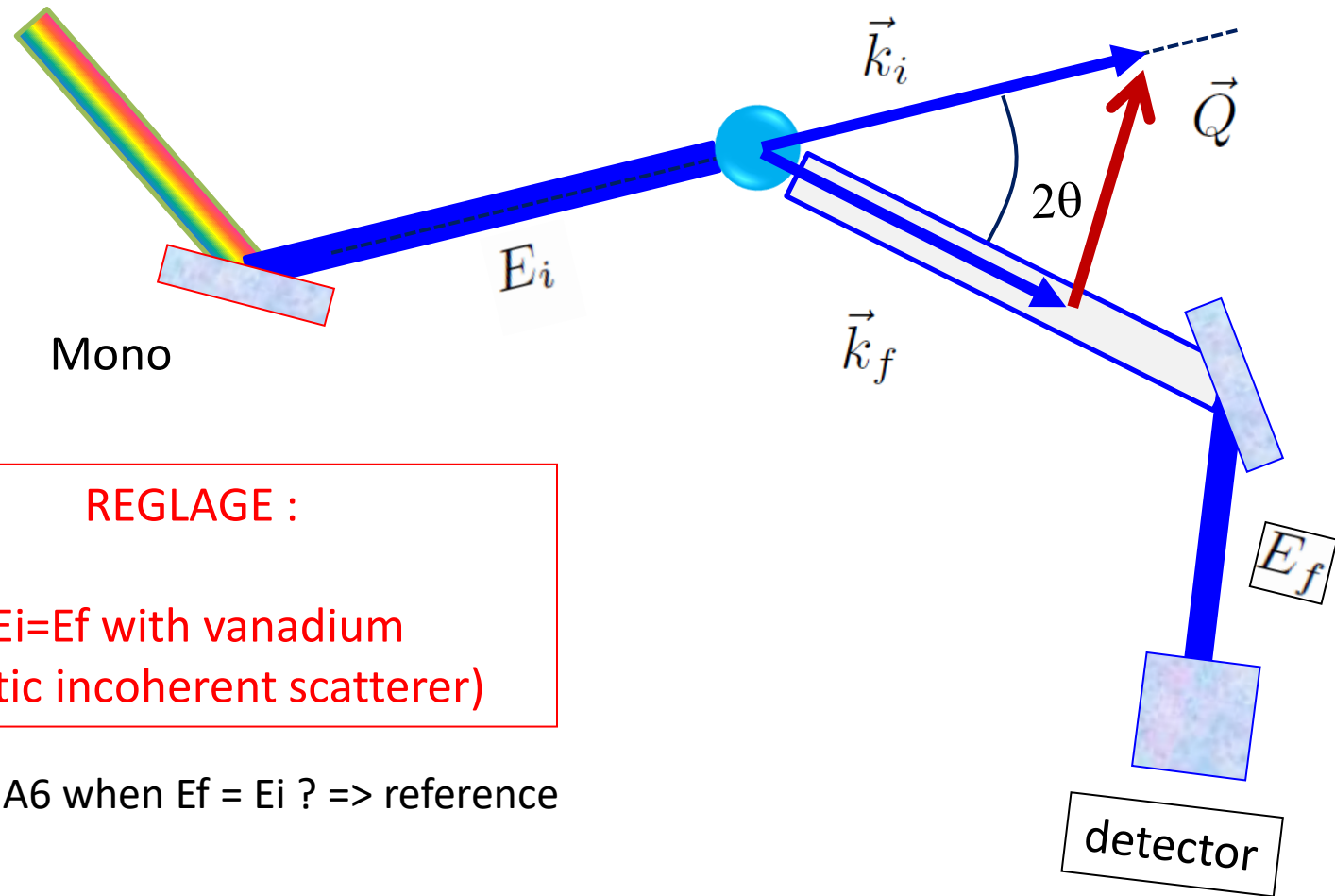






IN12





REGLAGE :

$E_i = E_f$ with vanadium
(elastic incoherent scatterer)

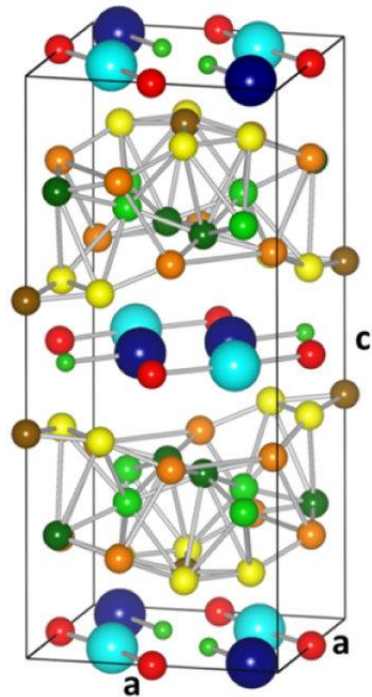
Ex : A5, A6 when $E_f = E_i$? $\Rightarrow$ reference

Sample II for tutorial : $\text{Nd}_2\text{Fe}_{14}\text{B}$

$$a = b = 8.8 \text{ \AA}$$

$$c = 12.2 \text{ \AA}$$

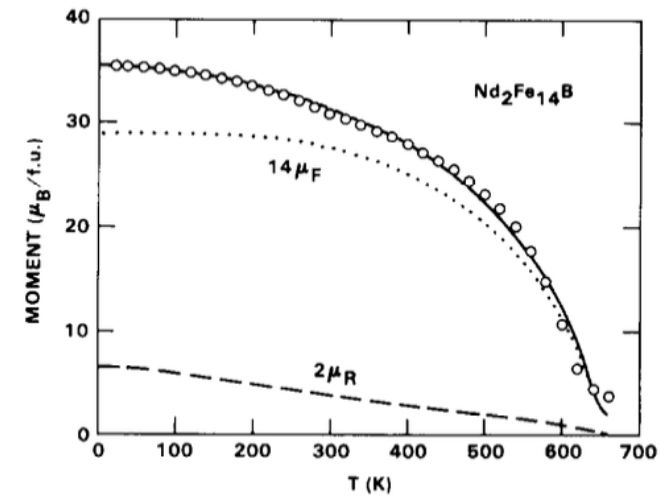
Space group : $P4_2/mnm$ $Z=4$

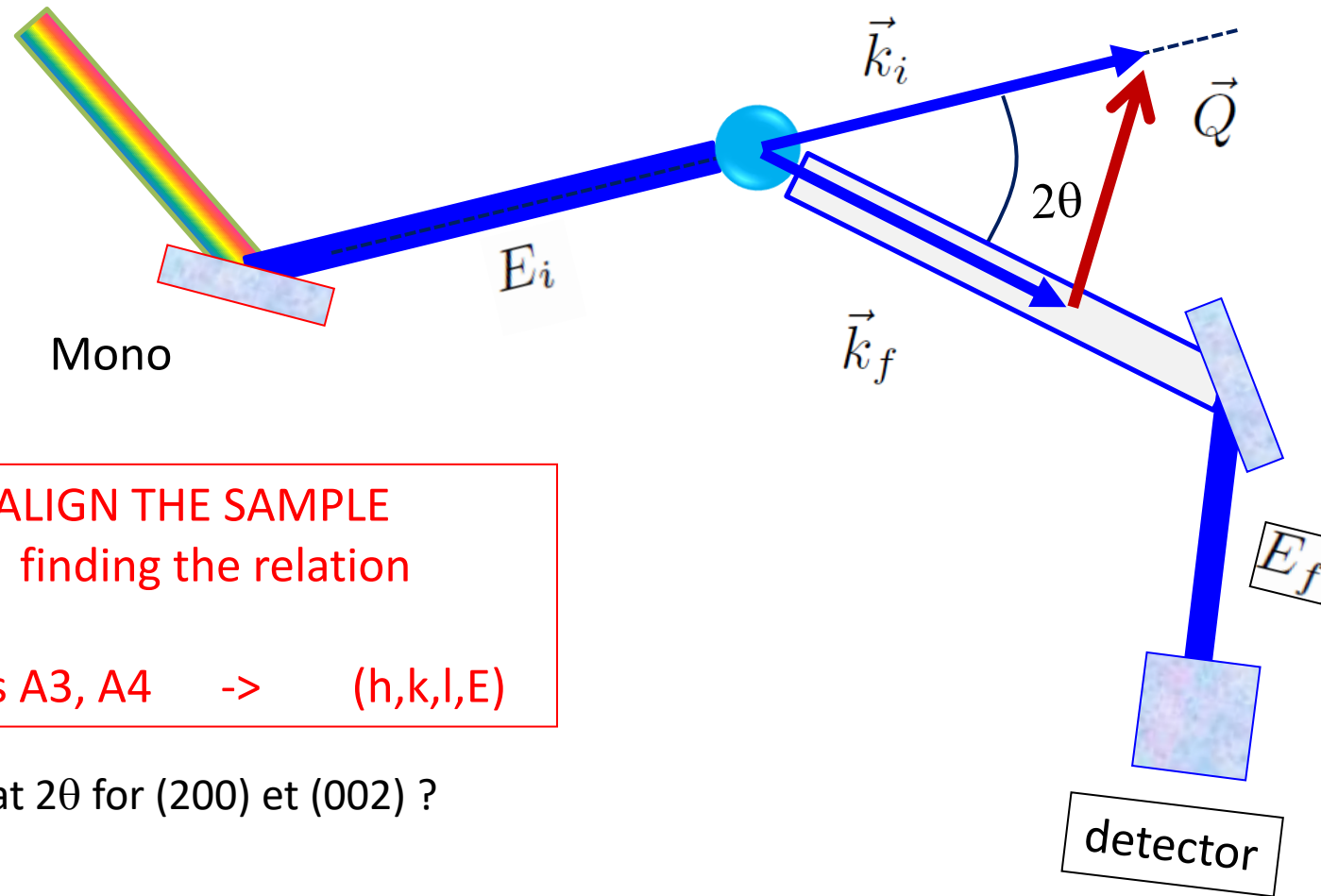


Atom	x	y	z
Nd(4f)	0.268	0.268	0.0
Nd(4g)	0.140	-0.140	0.0
Fe(16k ₁)	0.223	0.567	0.127
Fe(16k ₂)	0.037	0.360	0.176
Fe(8j ₁)	0.098	0.098	0.204
Fe(8j ₂)	0.317	0.317	0.246
Fe(4e)	0.5	0.5	0.114
Fe(4c)	0.0	0.5	0.0
B(4g)	0.371	-0.371	0.0

Magnetic ordering below $T_{\text{Curie}} \approx 585 \text{ K}$

Ferromagnetic m//c down to 135 K

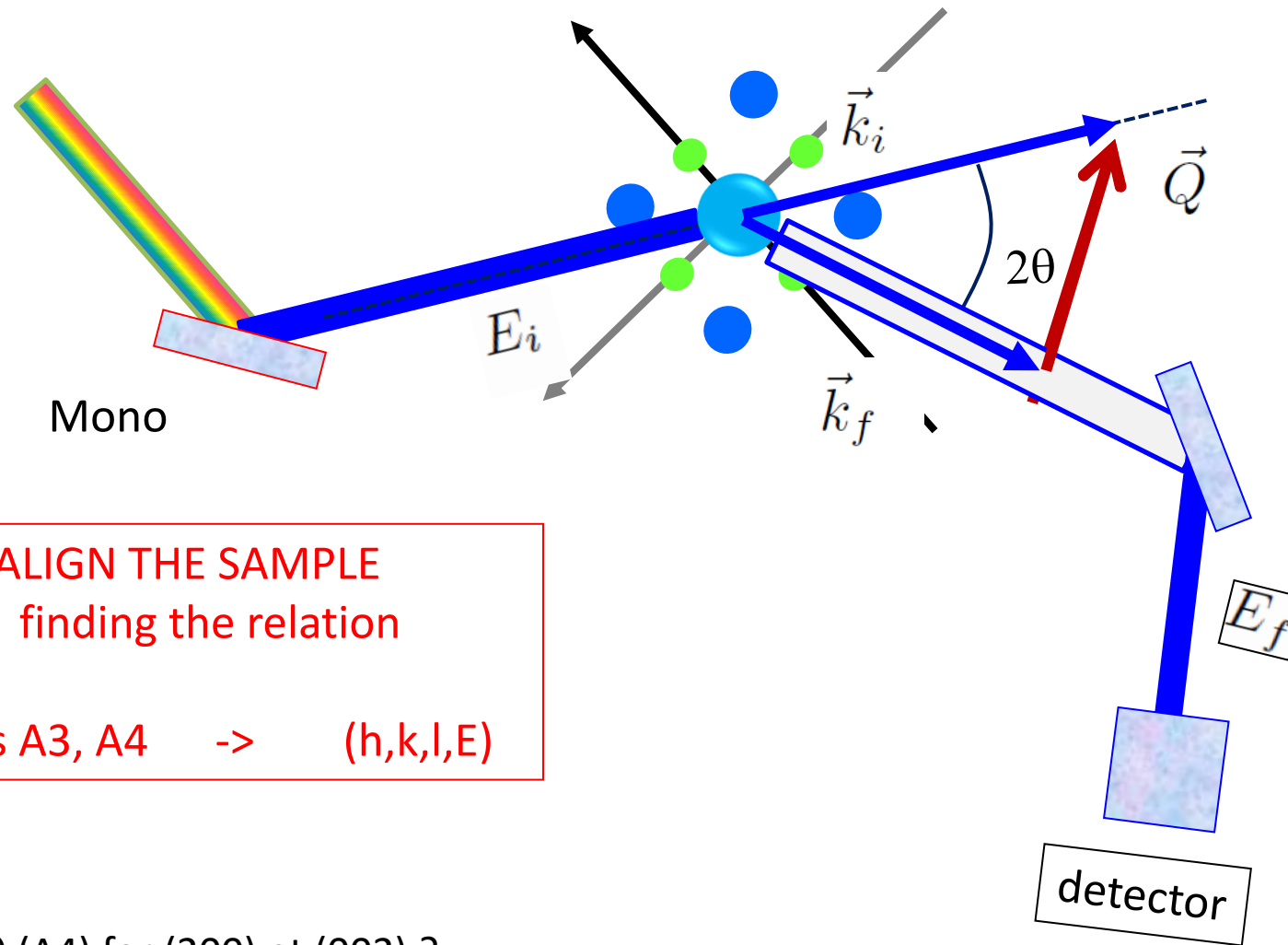




ALIGN THE SAMPLE
= finding the relation

angles $A_3, A_4 \rightarrow (h, k, l, E)$

Ex : what 2θ for (200) et (002) ?



ALIGN THE SAMPLE
= finding the relation

angles A3, A4 $\rightarrow$ (h,k,l,E)

Ex :

what 2θ (A4) for (200) et (002) ?

What sample rotation (A3) ?

PHYSICAL REVIEW B **102**, 014443 (2020)

Anisotropy and temperature dependence of the spin-wave stiffness in $\text{Nd}_2\text{Fe}_{14}\text{B}$: An inelastic neutron scattering investigation

H. Naser¹, C. Rado¹, G. Lapertot² and S. Raymond^{3,*}

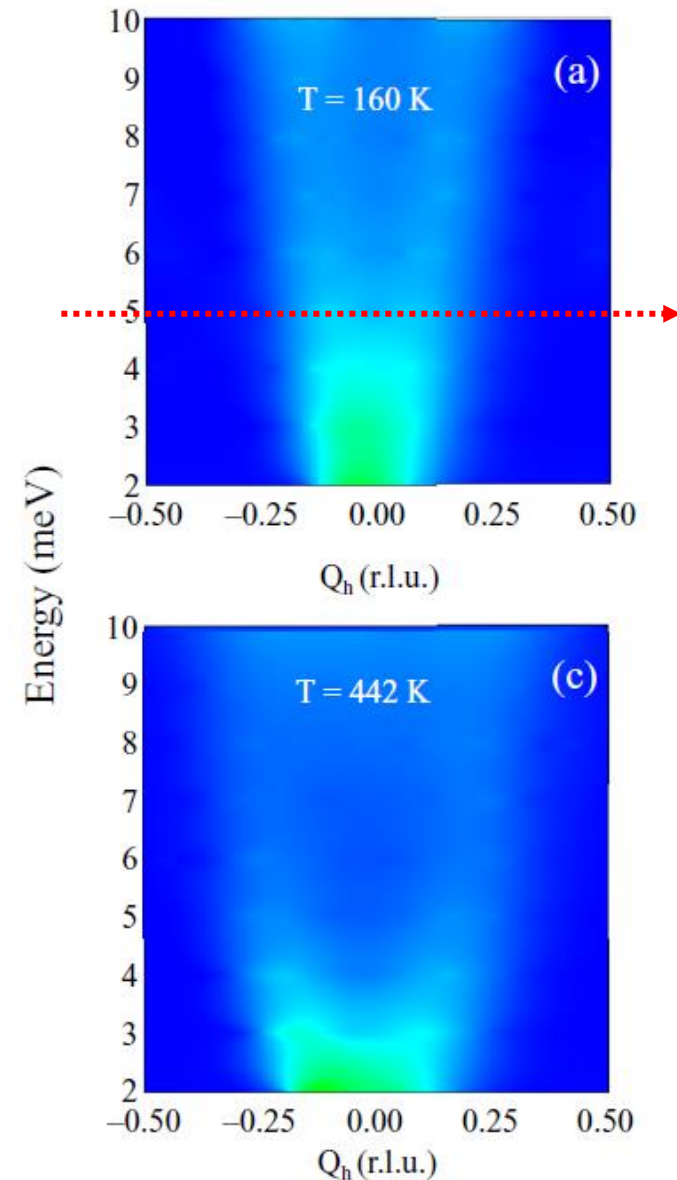
¹CEA, LITEN, Université Grenoble Alpes, 38000 Grenoble, France

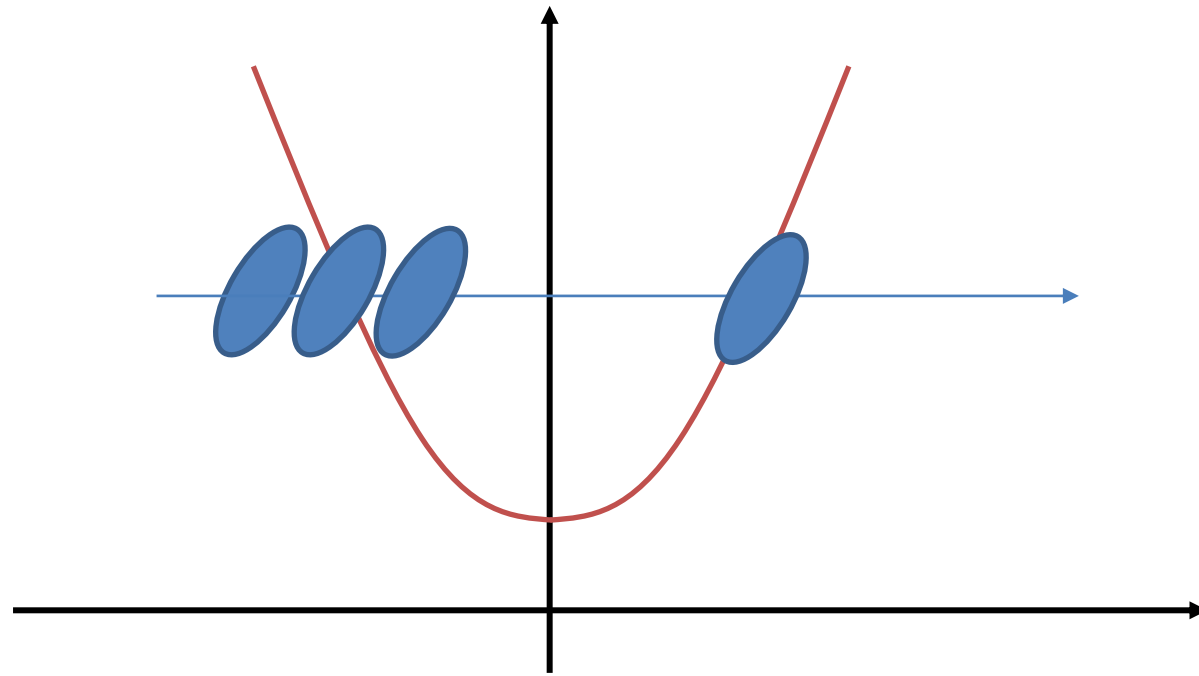
²Université Grenoble Alpes, CEA, IRIG, PHELIQS, IMAPEC, 38000 Grenoble, France

³Université Grenoble Alpes, CEA, IRIG, MEM, MDN, 38000 Grenoble, France

Determine
the spin wave dispersion relation
propagating along $(q_h 0 0)$
and stemming from the zone
center (002) .

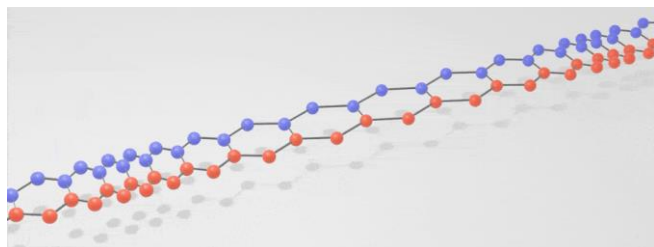
$D q_h^2$ law ?







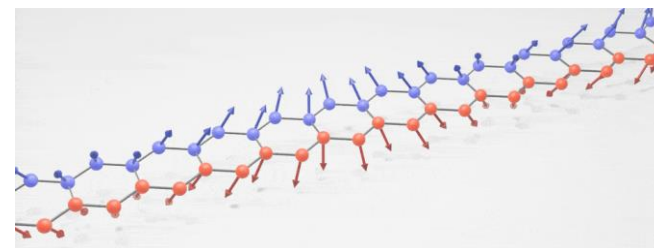
Lattice dynamics



Atomic force constants



Spin dynamics



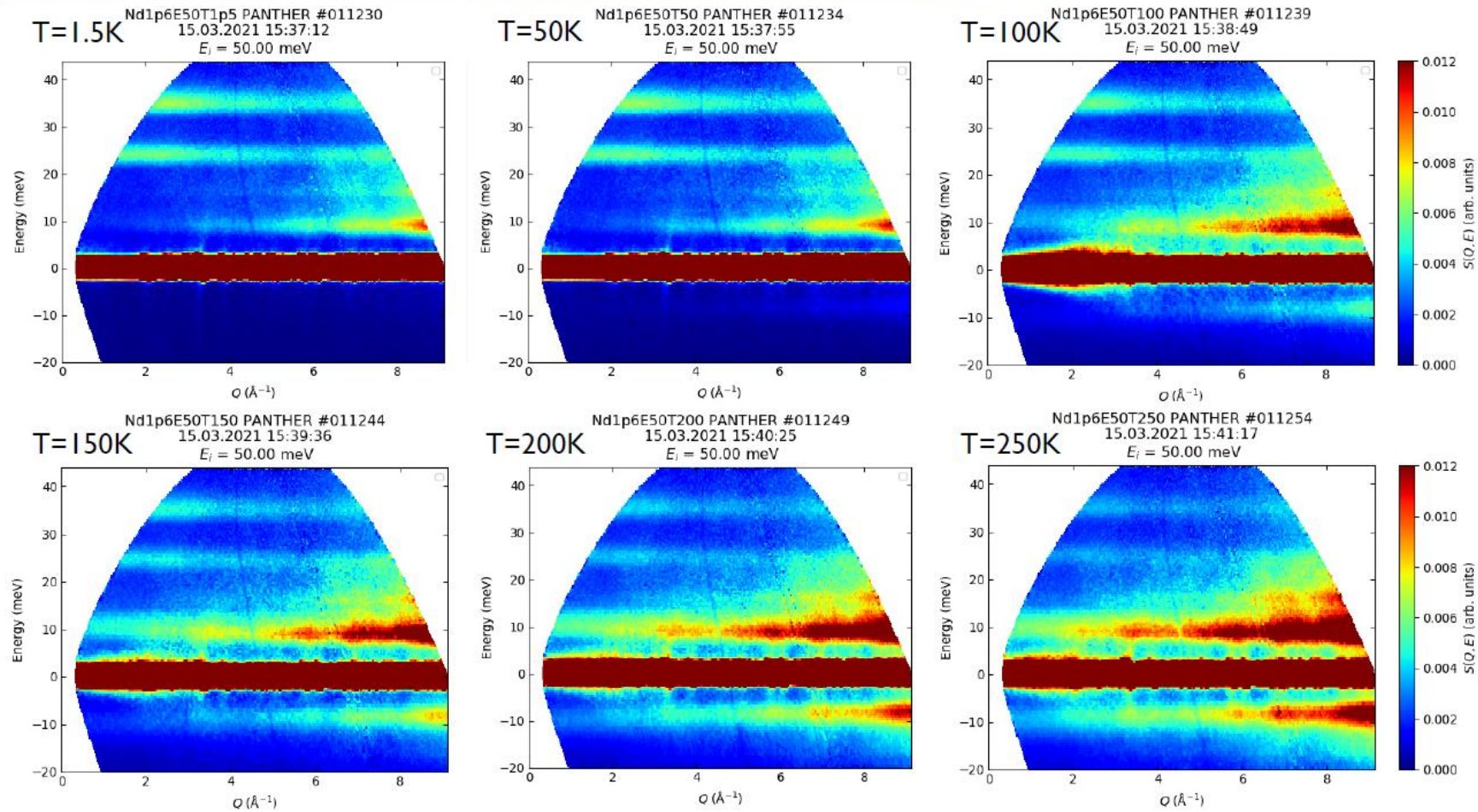
Magnetic exchange

THANK YOU FOR YOUR ATTENTION

8. More ...

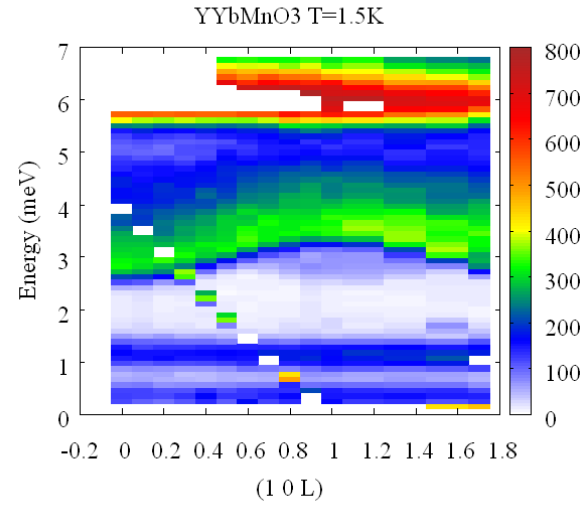
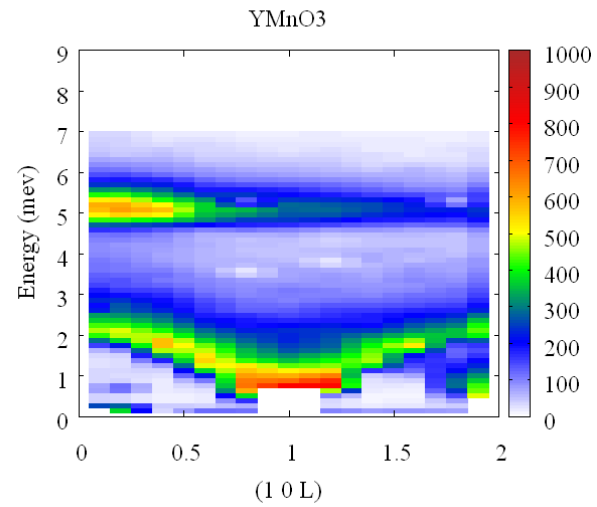


Crystal field

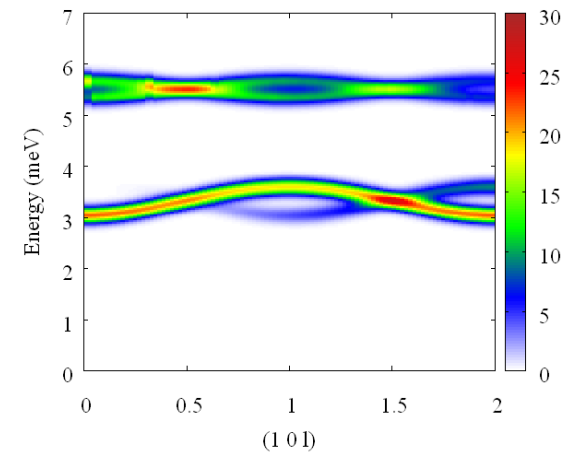
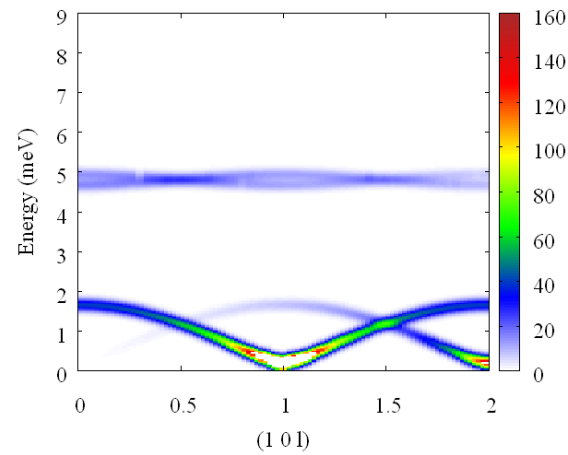


Triangular lattice

YMnO₃



YbMnO₃



Pyrochlore ferromagnet $\text{Yb}_2\text{Ti}_2\text{O}_7$

QUANTUM EXCITATIONS IN QUANTUM SPIN ICE

PHYS. REV. X **1**, 021002 (2011)

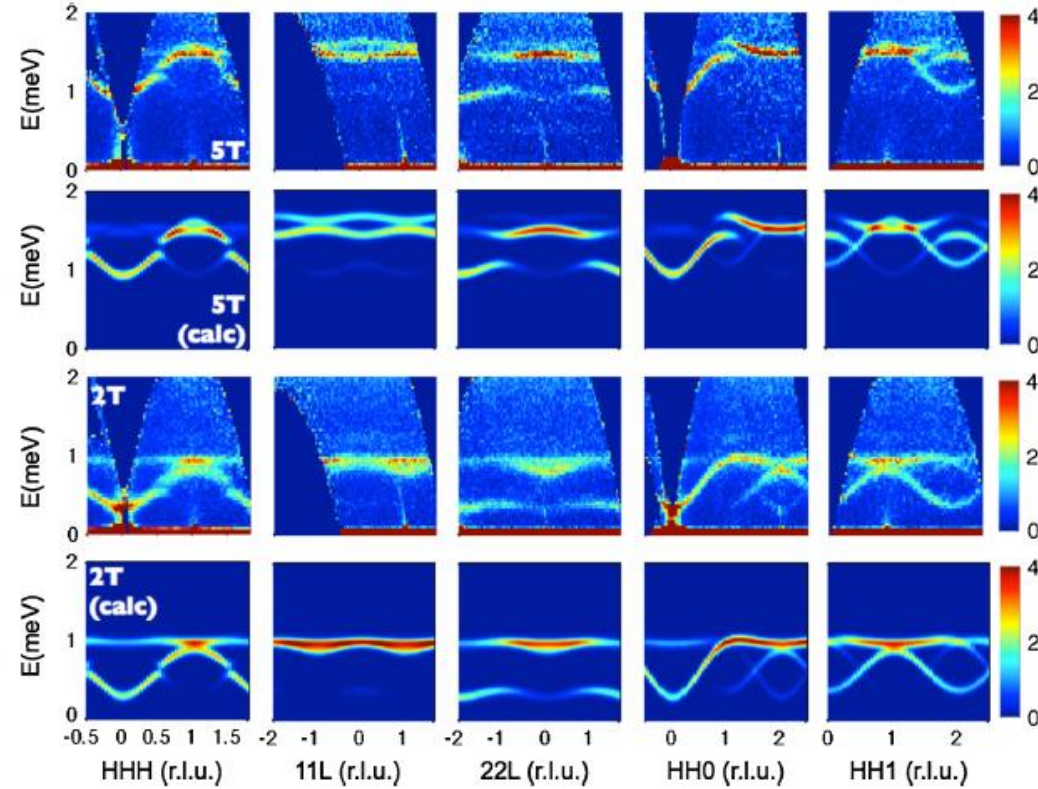


FIG. 1. The measured $S(\mathbf{Q}, \omega)$ at $T = 30$ mK, sliced along various directions in the HHL plane, for both $H = 5$ T (first row) and $H = 2$ T (third row). The second and fourth rows show the calculated spectrum for these two field strengths, based on an anisotropic exchange model with five free parameters (see text) that were extracted by fitting to the 5 T data set. For a realistic comparison to the data, the calculated $S(\mathbf{Q}, \omega)$ is convoluted with a Gaussian of full-width 0.09 meV. Both the 2 T and 5 T data sets, composed of spin wave dispersions along five different directions, are described extremely well by the same parameters. (Note that r.l.u. stands for reciprocal lattice units.)

Pyrochlore antiferromagnet $\text{Er}_2\text{Ti}_2\text{O}_7$

PRL **109**, 167201 (2012)

PHYSICAL REVIEW LETTERS

week ending
19 OCTOBER 2012

Order by Quantum Disorder in $\text{Er}_2\text{Ti}_2\text{O}_7$

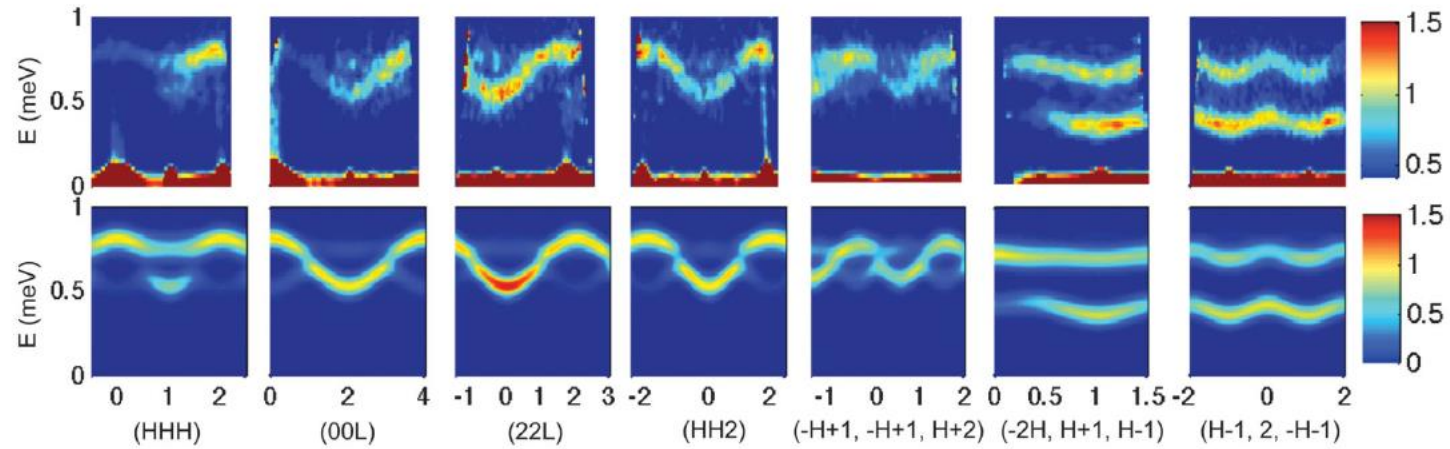
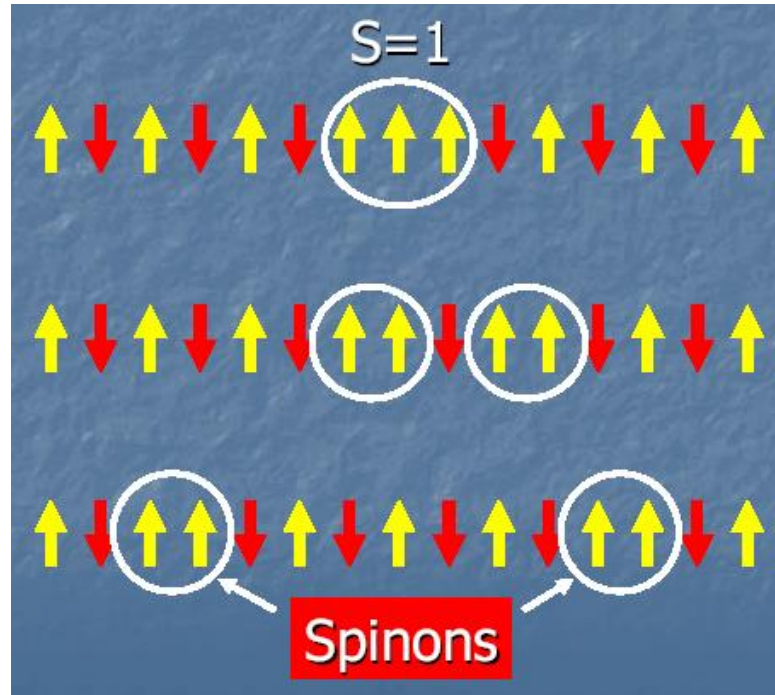


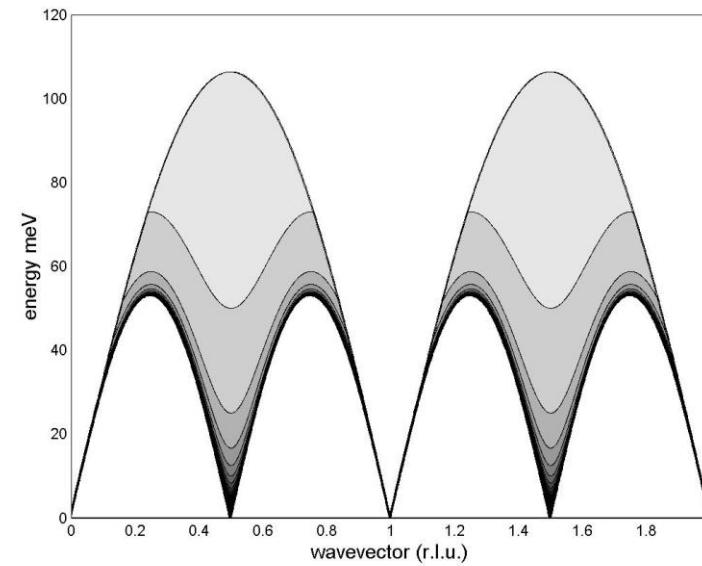
FIG. 1 (color). The measured $S(\mathbf{Q}, \omega)$ at $T = 30$ mK, $H = 3$ T sliced along several directions. The first five columns show $S(\mathbf{Q}, \omega)$ in the HHL plane, with the field applied along $[1\bar{1}0]$, while the last two columns show $S(\mathbf{Q}, \omega)$ for the field along $[111]$. Top row: measured $S(\mathbf{Q}, \omega)$. Bottom row: calculated $S(\mathbf{Q}, \omega)$, based on an anisotropic exchange model with six free parameters (see text) that were extracted by fitting to the measured dispersions.

Quantum magnets

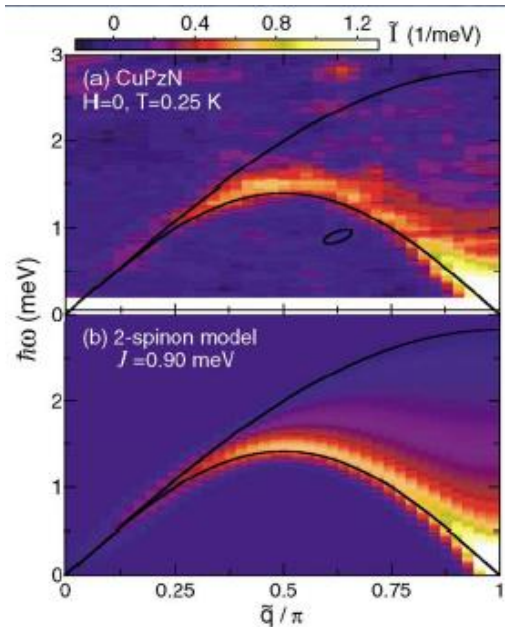
Spin waves fractionalize into spinons (also called deconfined excitations)



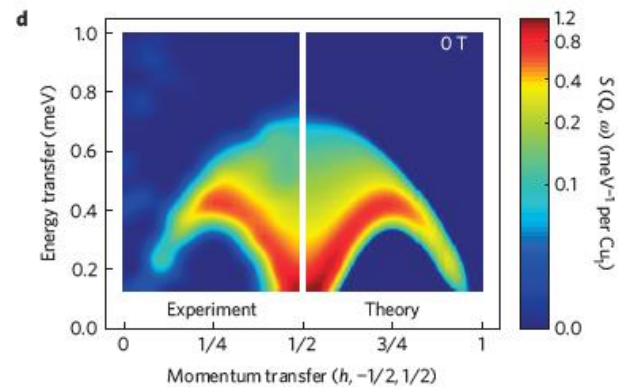
Cartoon by F. Mila



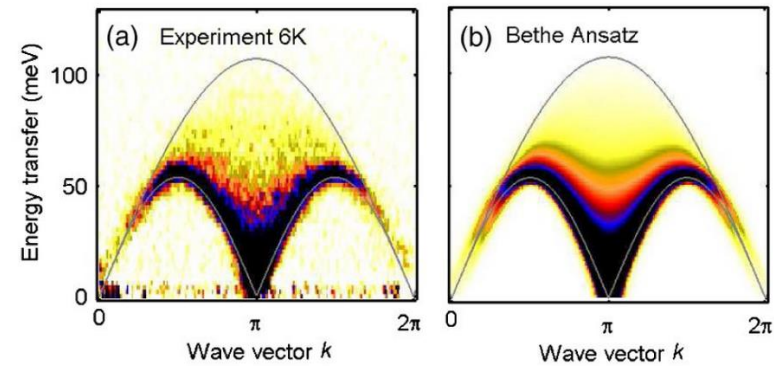
Quantum magnets



CuPzN, Stone et al, PRL 2003



CuSO₄ 5D₂O Mourigal et al, Nature Physics 2013



KCuF₃, Lake et al, Nature Materials 2005

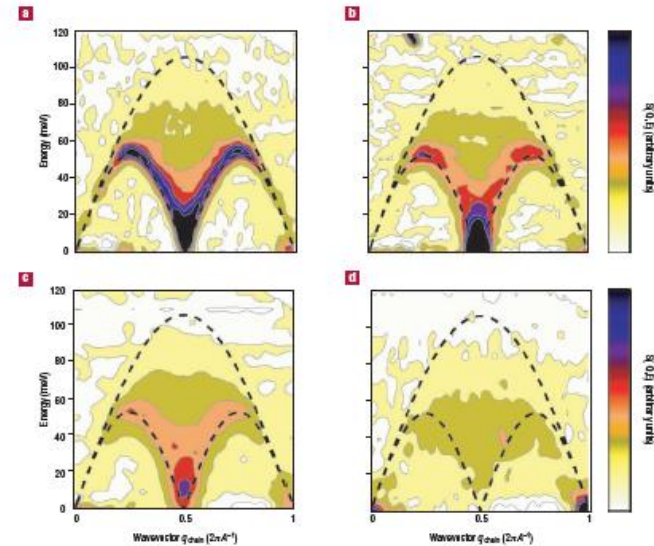
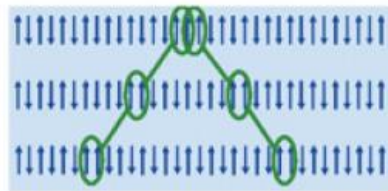
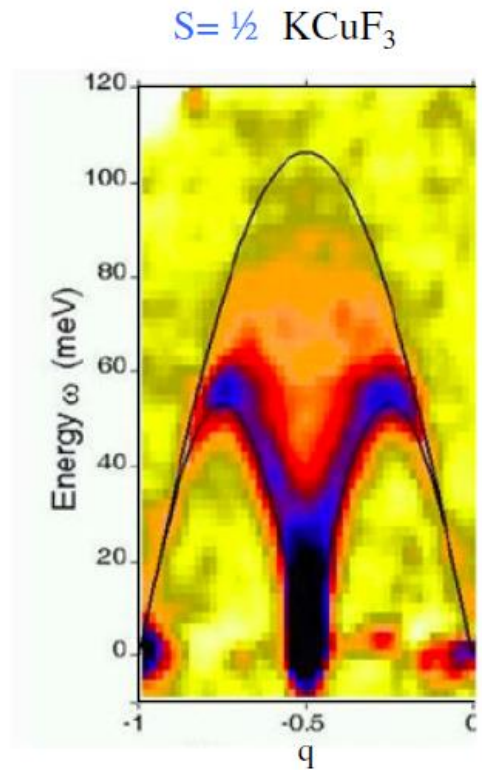
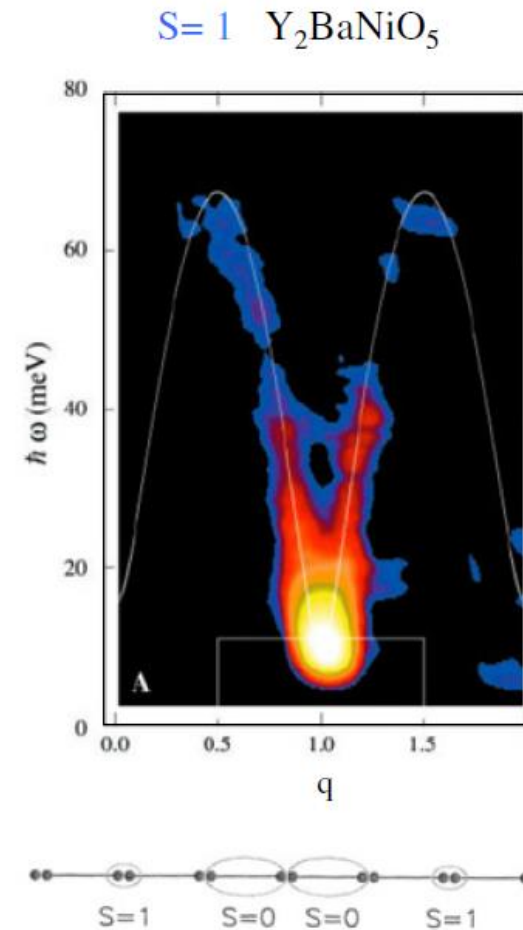


Figure 1 Inelastic neutron scattering data for KCuF₃. The data is plotted as a function of E and q parallel to the chains for the temperatures a, $T=6$ K, b, $T=50$ K, c, $T=150$ K and d, $T=300$ K. The colours indicate the size of the neutron scattering cross-section $S(Q, E)$ and the superimposed black dashed lines indicate the region where the multi-spinon continuum is predicted at $T=0$ K by the Bethe Ansatz equation (1). The data was collected using the MAPS time-of-flight spectrometer at ISIS, Rutherford Appleton Laboratory, UK.

Quantum magnets



Spin correlations decay as power law
Spin $\frac{1}{2}$ particles : continuum
gapless



Spin correlations decay exponentially
Spin 1 excitation : well defined mode
(Haldane) Gapped spectrum

The susceptibility (hence the correlations) are sensitive to nesting properties of the Fermi surface

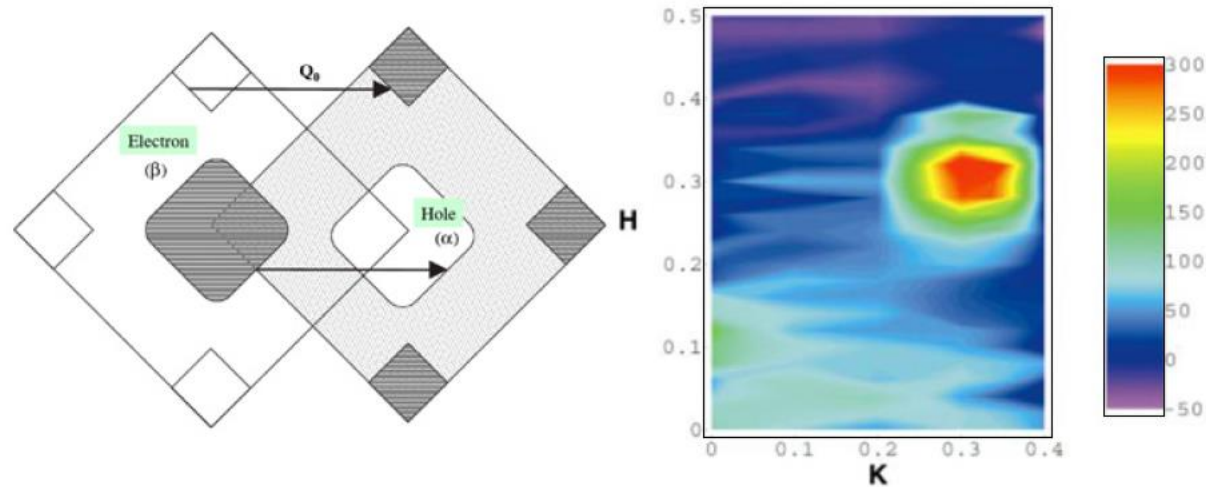
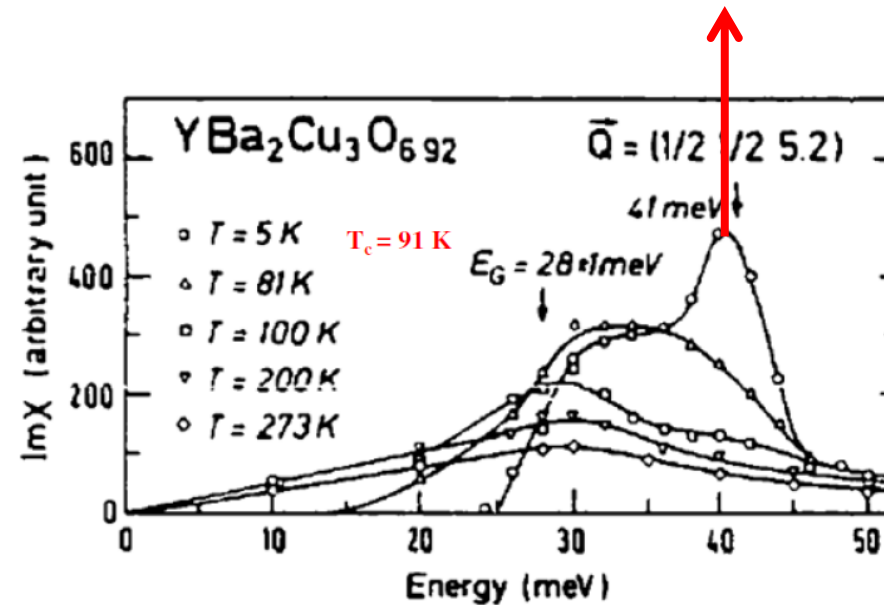


Figure 10. Left: sketch of the nesting feature between α and β bands of Sr_2RuO_4 . Right: (H, K) map of the magnetic excitations in Sr_2RuO_4 at 10 K for an energy transfer of 4.1 meV [21].

Interactions may create a bound state (spin exciton)
Resonance peak in High Tc superconductors and heavy fermions ?

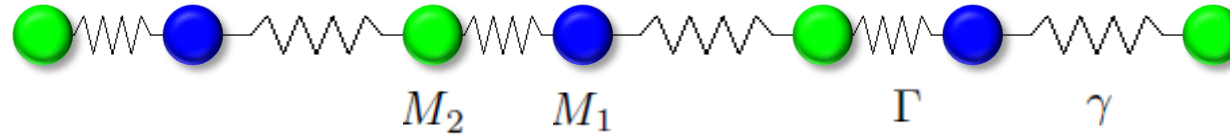
$$\chi_0(\mathbf{q}, \omega) = -2\mu_B^2 \sum_{\mathbf{k}} \frac{f_{\mathbf{k}+\mathbf{q},\sigma'} - f_{\mathbf{k},\sigma}}{\epsilon_{\mathbf{k}+\mathbf{q},\sigma'} - \epsilon_{\mathbf{k},\sigma} - \hbar\omega + i\epsilon}$$

$$\chi_0(\mathbf{q}, \omega) = -2\mu_B^2 \sum_{\mathbf{k}} \left(1 - \frac{\Delta_{\mathbf{k}}\Delta_{\mathbf{k}+\mathbf{q}} + \epsilon_{\mathbf{k}}\epsilon_{\mathbf{k}+\mathbf{q}}}{E_{\mathbf{k}}E_{\mathbf{k}+\mathbf{q}}} \right) \frac{f_{\mathbf{k}+\mathbf{q},\sigma'} + f_{\mathbf{k},\sigma} - 1}{E_{\mathbf{k}+\mathbf{q},\sigma'} - E_{\mathbf{k},\sigma} - \hbar\omega + i\epsilon}.$$



Example 2 : Phonons

Di-atomic chain



$$M_1 \frac{d^2}{dt^2} u_{m,1} = \Gamma (u_{m,2} - u_{m,1}) + \gamma (u_{m-1,2} - u_{m,1})$$

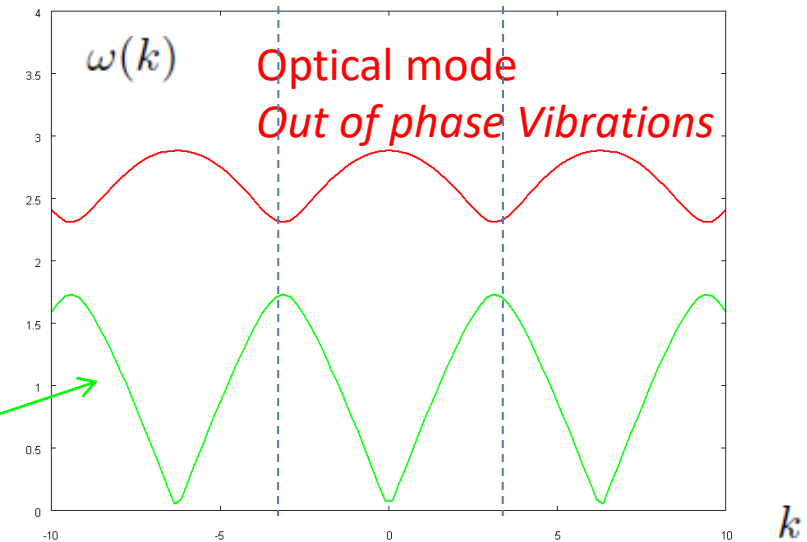
$$M_2 \frac{d^2}{dt^2} u_{m,2} = \Gamma (u_{m,1} - u_{m,2}) + \gamma (u_{m+1,1} - u_{m,2})$$

Force constants
between atoms

Fourier transform: for each k , eigenvalue problem

$$\begin{pmatrix} \omega^2 - \frac{\Gamma + \gamma}{M_1} & \frac{\Gamma + \gamma e^{-ika}}{\sqrt{M_1 M_2}} \\ \frac{\Gamma + \gamma e^{ika}}{\sqrt{M_1 M_2}} & \omega^2 - \frac{\Gamma + \gamma}{M_2} \end{pmatrix} \begin{pmatrix} \sqrt{M_1} u_1 \\ \sqrt{M_2} u_2 \end{pmatrix} = 0$$

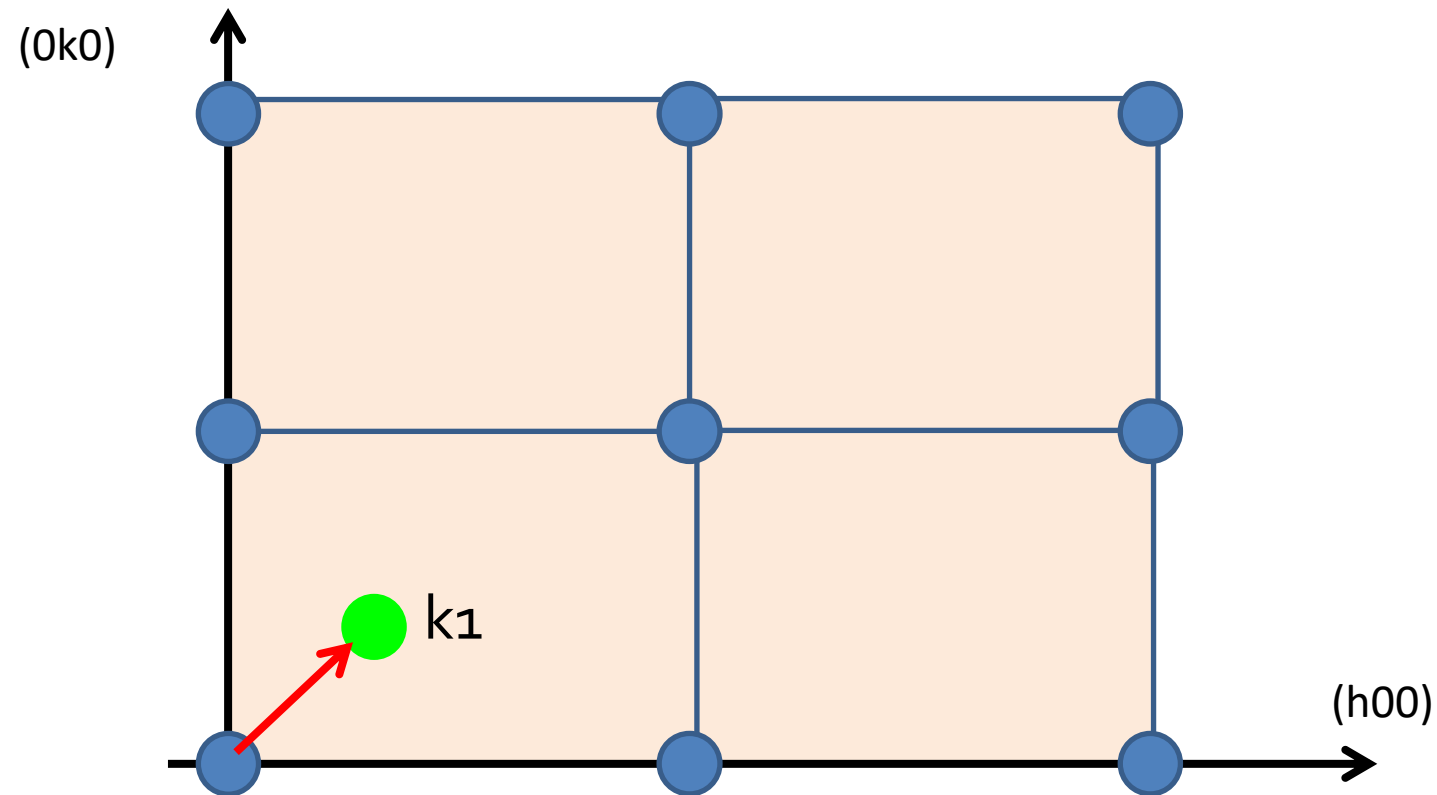
Acoustic mode
In phase Vibrations



Example 2 : Phonons

In real material :

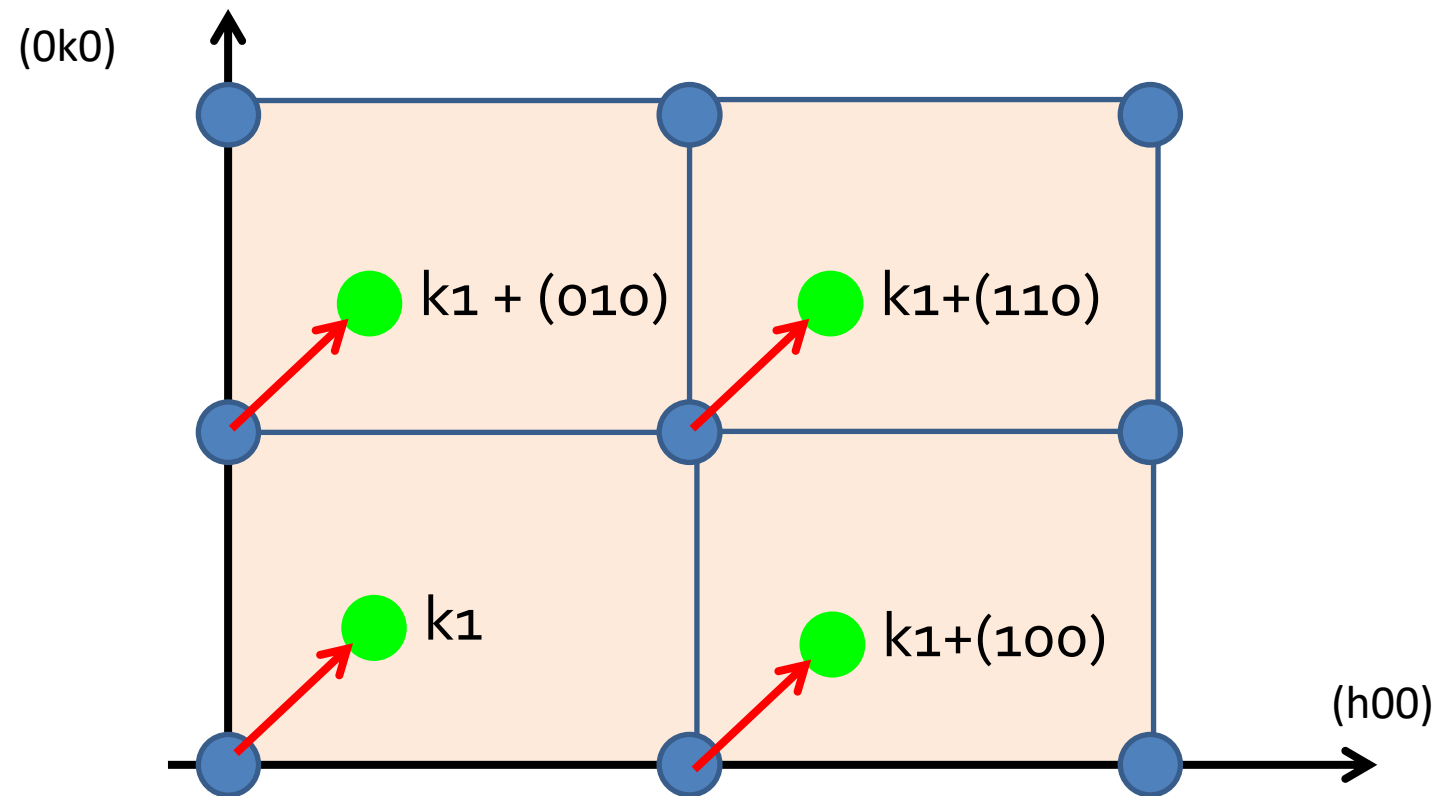
- 3N branches (3 acoustic)
- periodic dispersion (Brillouin zone, lattice invariance)



Example 2 : Phonons

In real material :

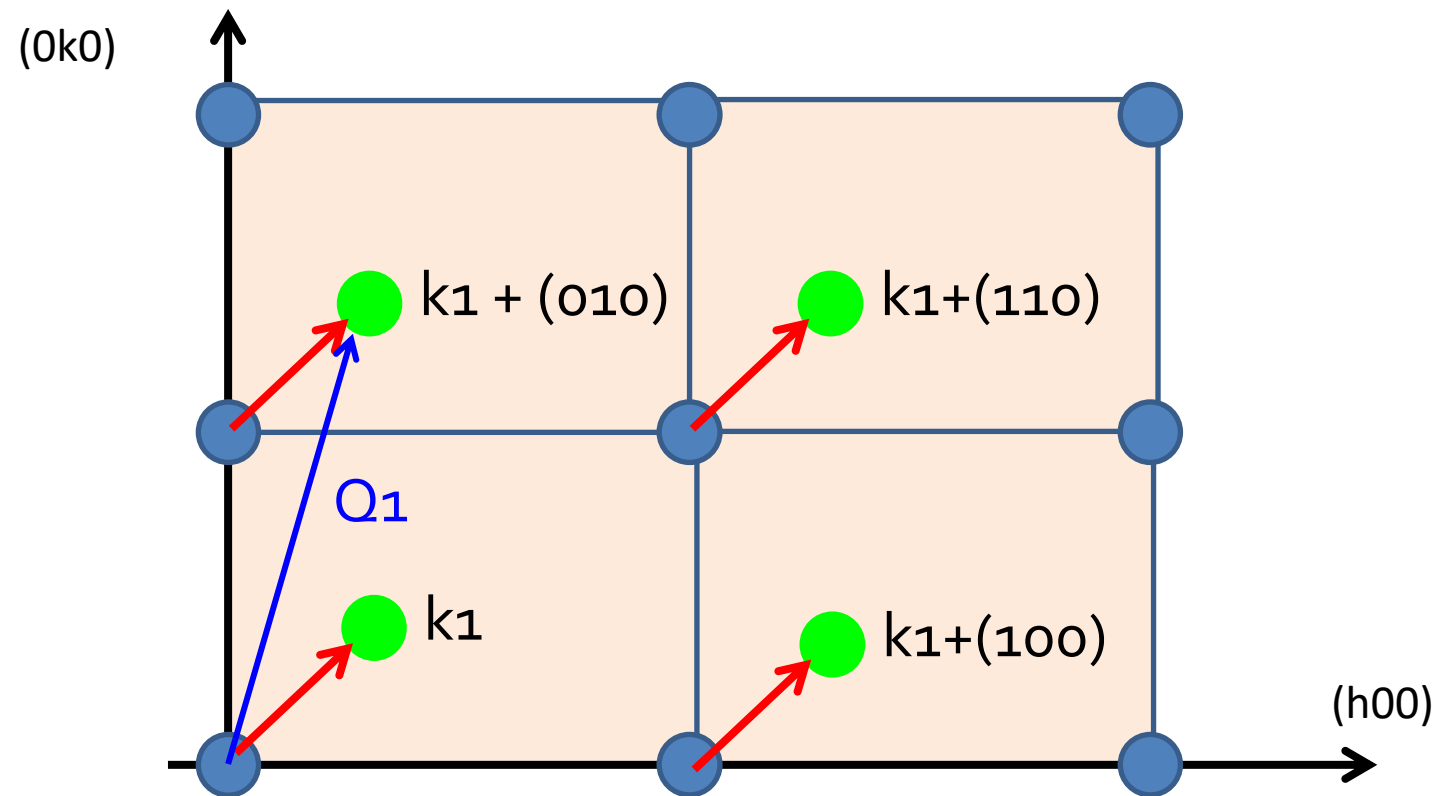
- 3N branches (3 acoustic)
- periodic dispersion (Brillouin zone, lattice invariance)



Example 2 : Phonons

In real material :

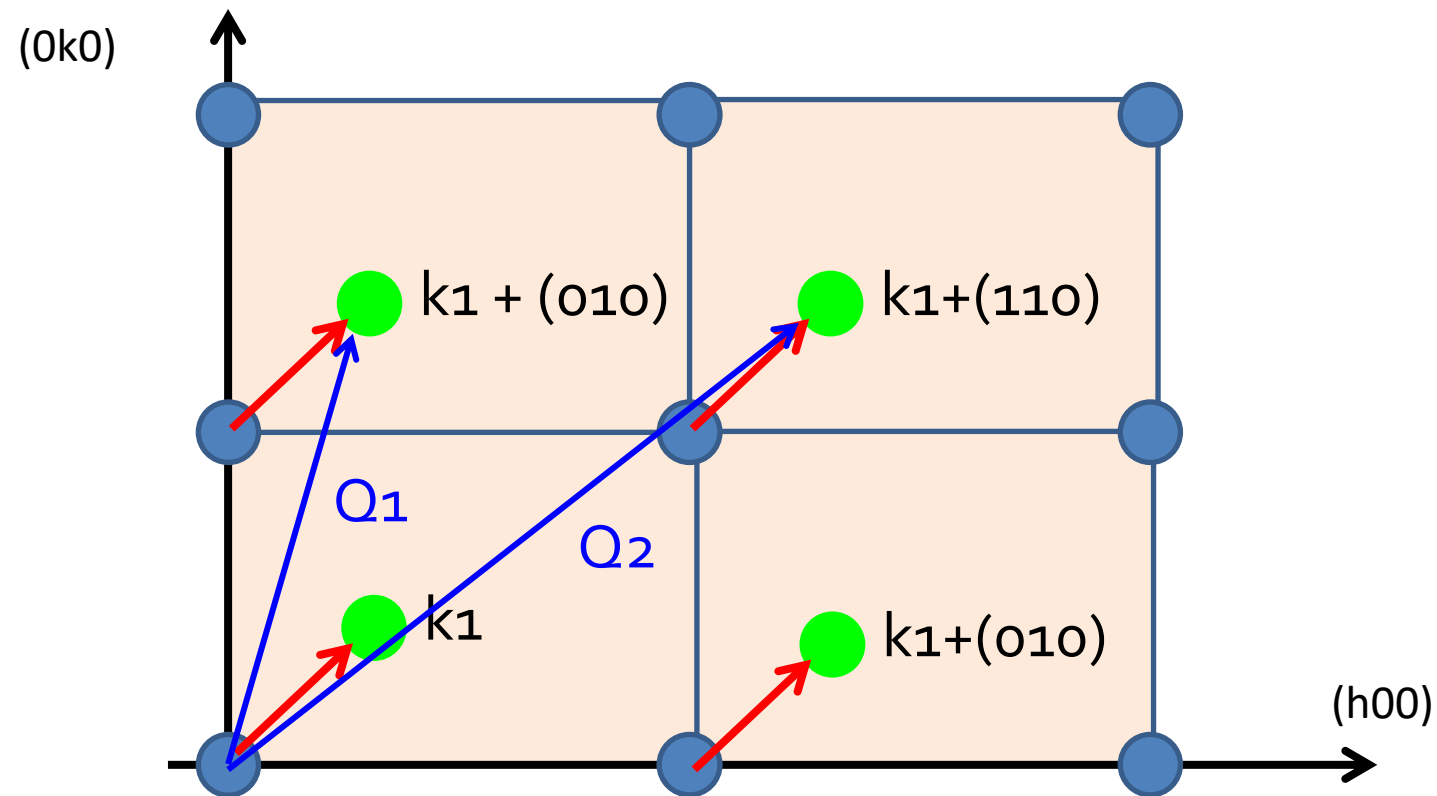
- 3N branches (3 acoustic)
- periodic dispersion (Brillouin zone, lattice invariance)



Example 2 : Phonons

In real material :

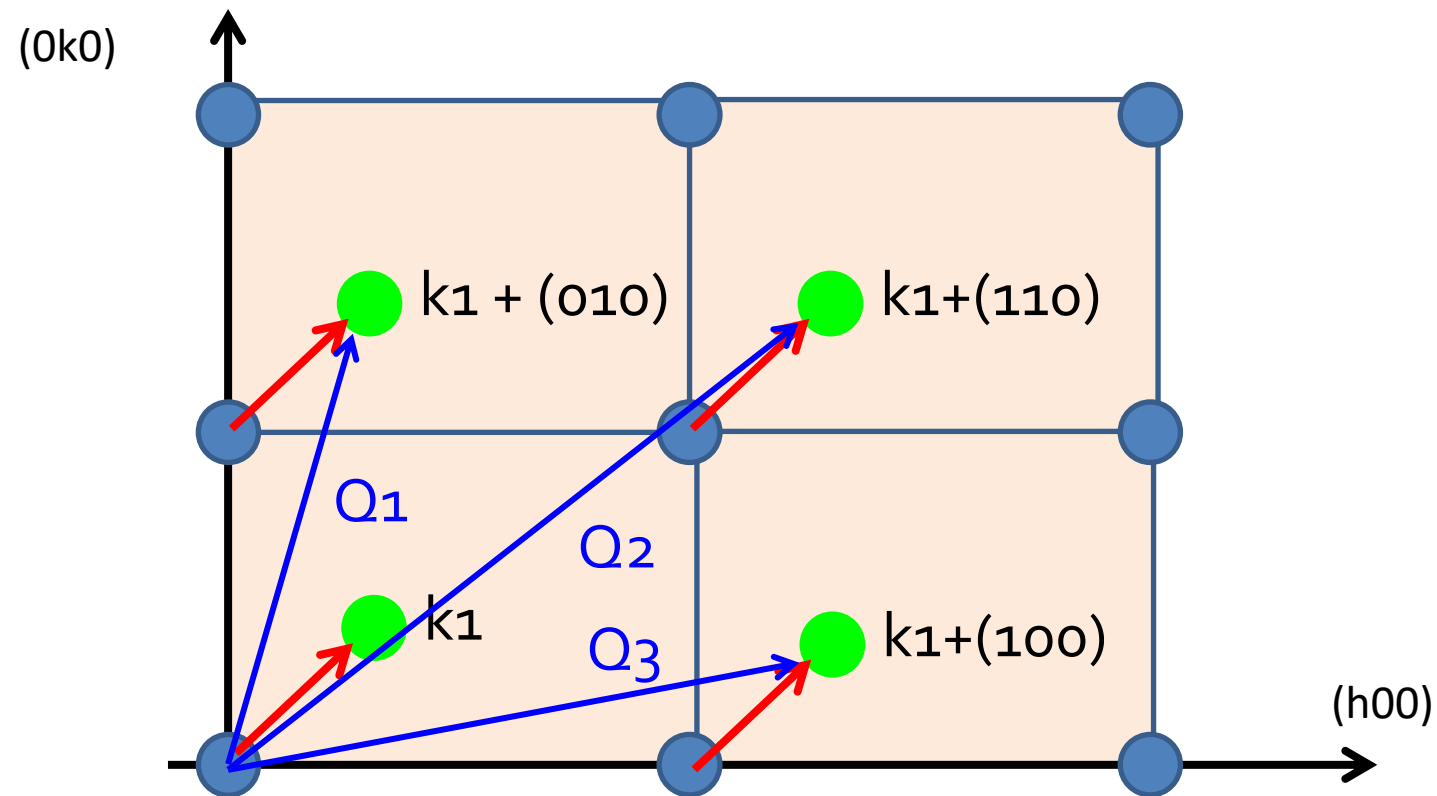
- 3N branches (3 acoustic)
- periodic dispersion (Brillouin zone, lattice invariance)

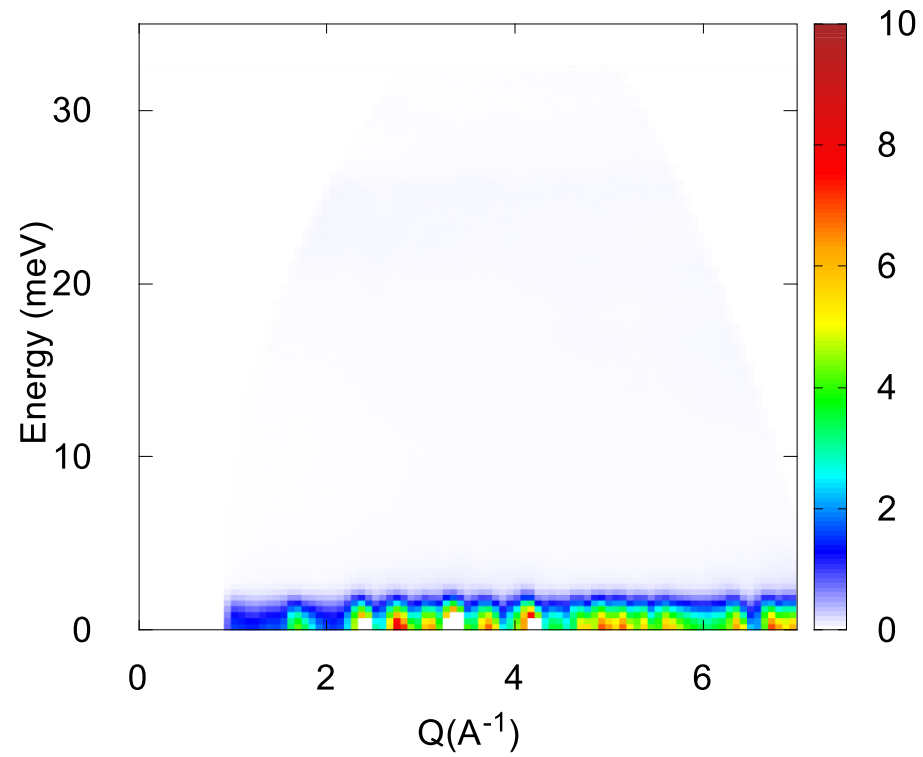


Example 2 : Phonons

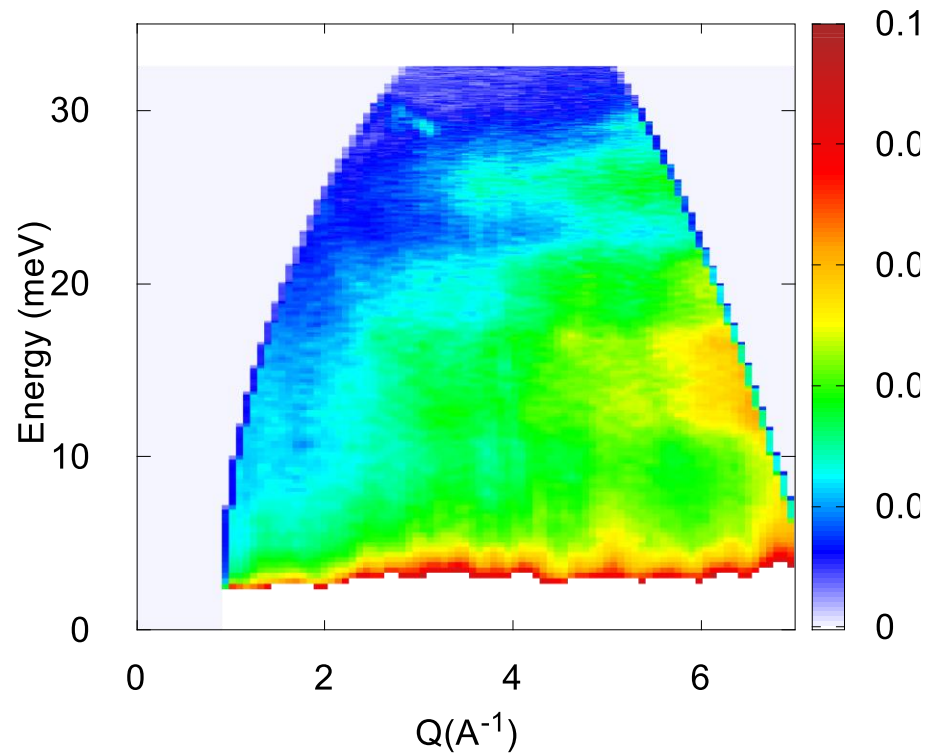
In real material :

- 3N branches (3 acoustic)
- periodic dispersion (Brillouin zone, lattice invariance)





- YNiO_3 powder sample
 $T = 200 \text{ K}$
- Not much to see,
except the elastic line



- YNiO₃ powder sample
T = 200 K

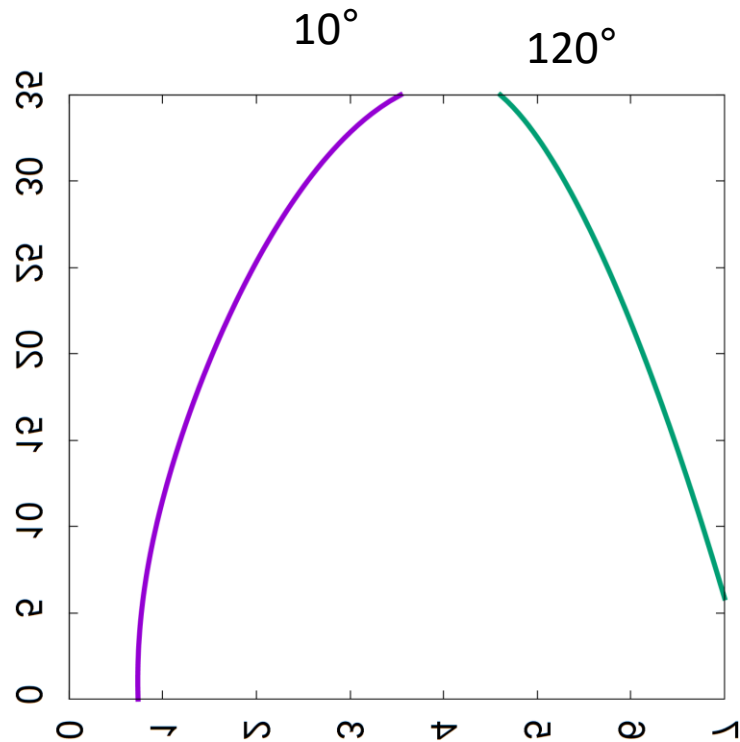
- Not much to see,
except the elastic line

Change color-scale !!

- Enveloppes

$$E = \hbar\omega = E_i - E_f$$

$$\vec{Q} = \vec{k}_i - \vec{k}_f$$

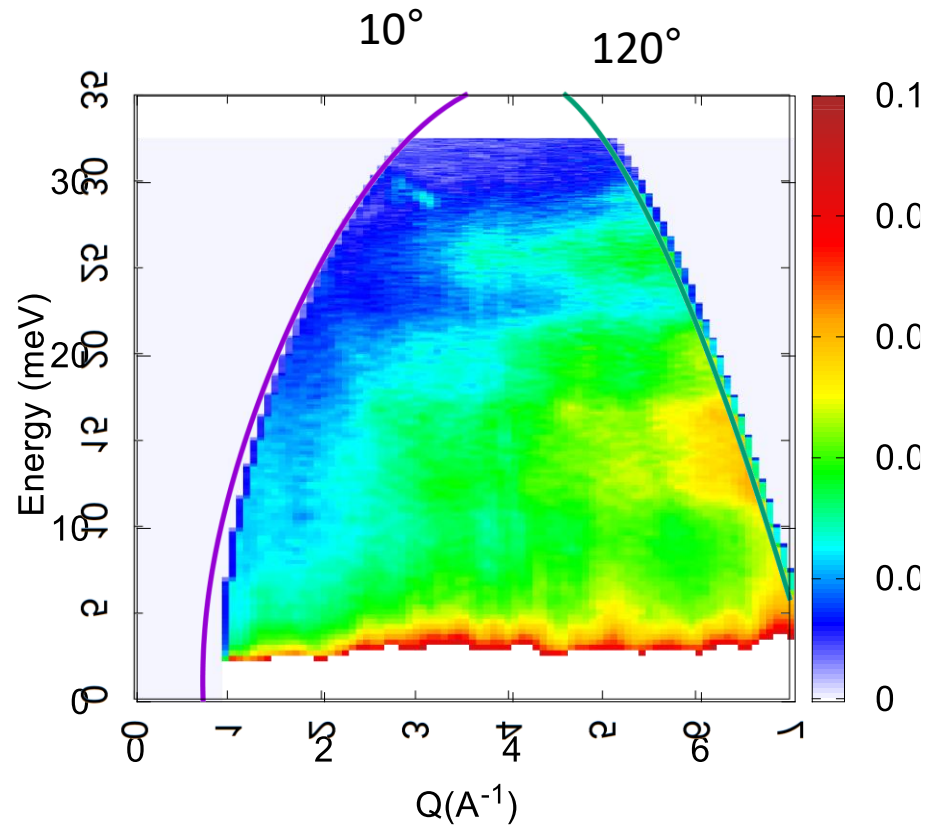


○ Enveloppes $\vec{Q} = \vec{k}_i - \vec{k}_f$

$$Q^2 = k_i^2 + k_f^2 - 2k_i k_f \cos 2\theta$$

$$\frac{\hbar^2 Q^2}{2m} = E_i + E_f - 2\sqrt{E_i E_f} \cos 2\theta$$

$$\frac{\hbar^2 Q^2}{2m} = 2E_i - E - 2\sqrt{E_i(E_i - E)} \cos 2\theta$$



○ Enveloppes $\vec{Q} = \vec{k}_i - \vec{k}_f$

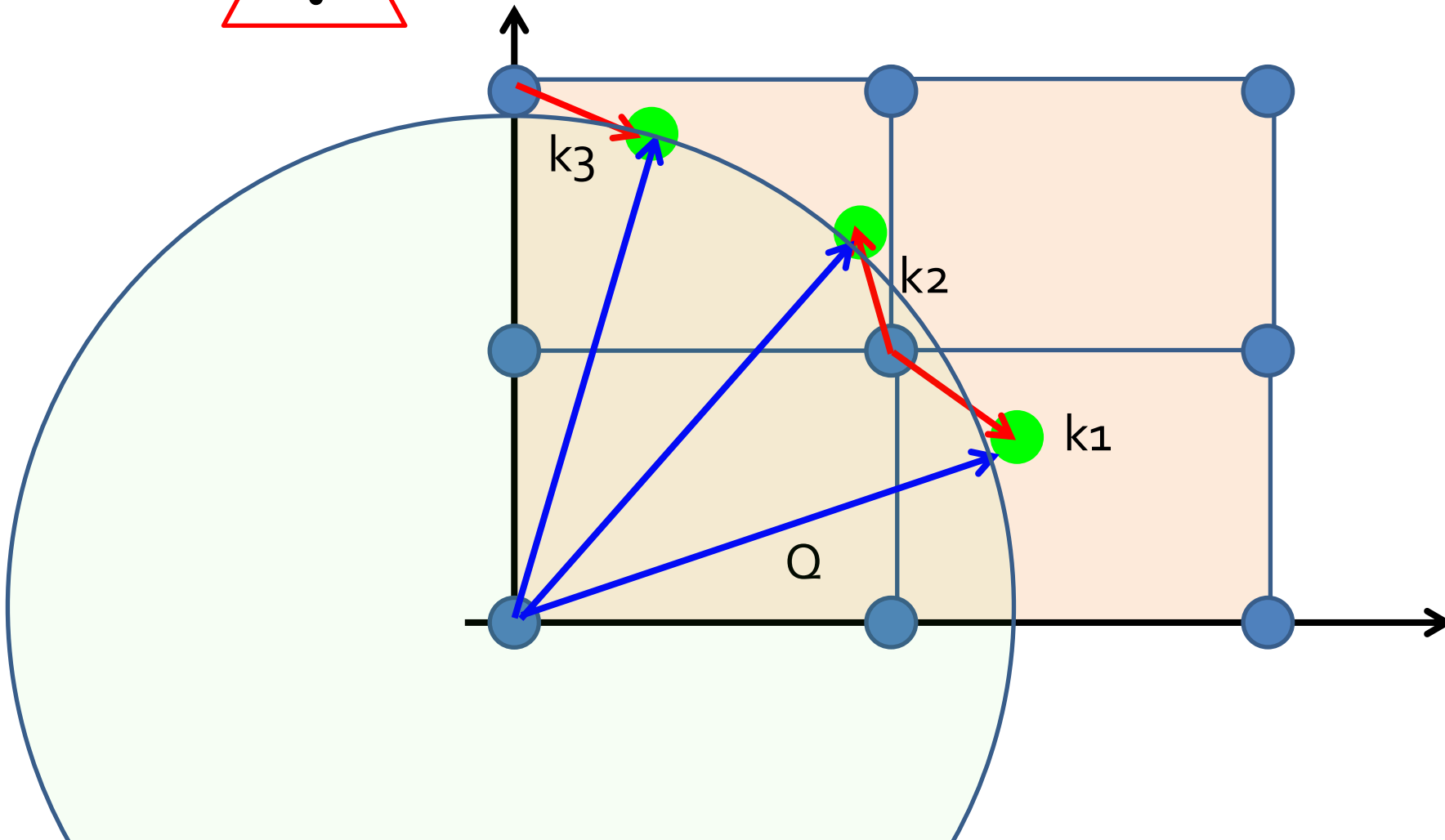
$$Q^2 = k_i^2 + k_f^2 - 2k_i k_f \cos 2\theta$$

$$\frac{\hbar^2 Q^2}{2m} = E_i + E_f - 2\sqrt{E_i E_f} \cos 2\theta$$

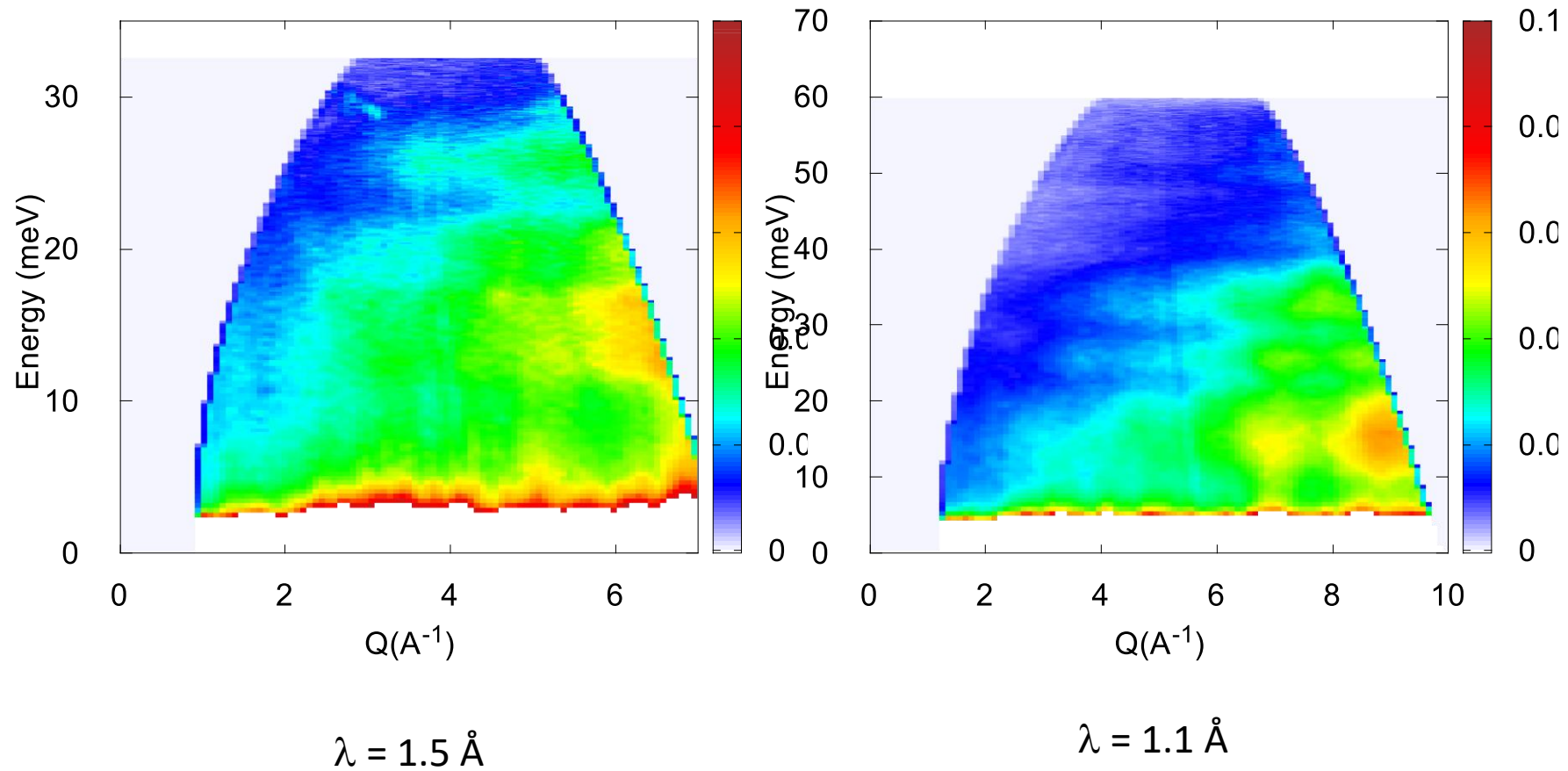
$$\frac{\hbar^2 Q^2}{2m} = 2E_i - E - 2\sqrt{E_i(E_i - E)} \cos 2\theta$$



Because of the powder average, different phonon states with different k 's but same modulus of Q are summed together, leading to a *partial density of states*

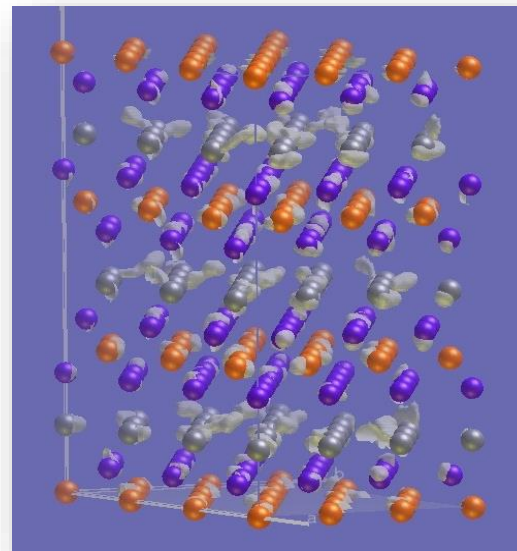
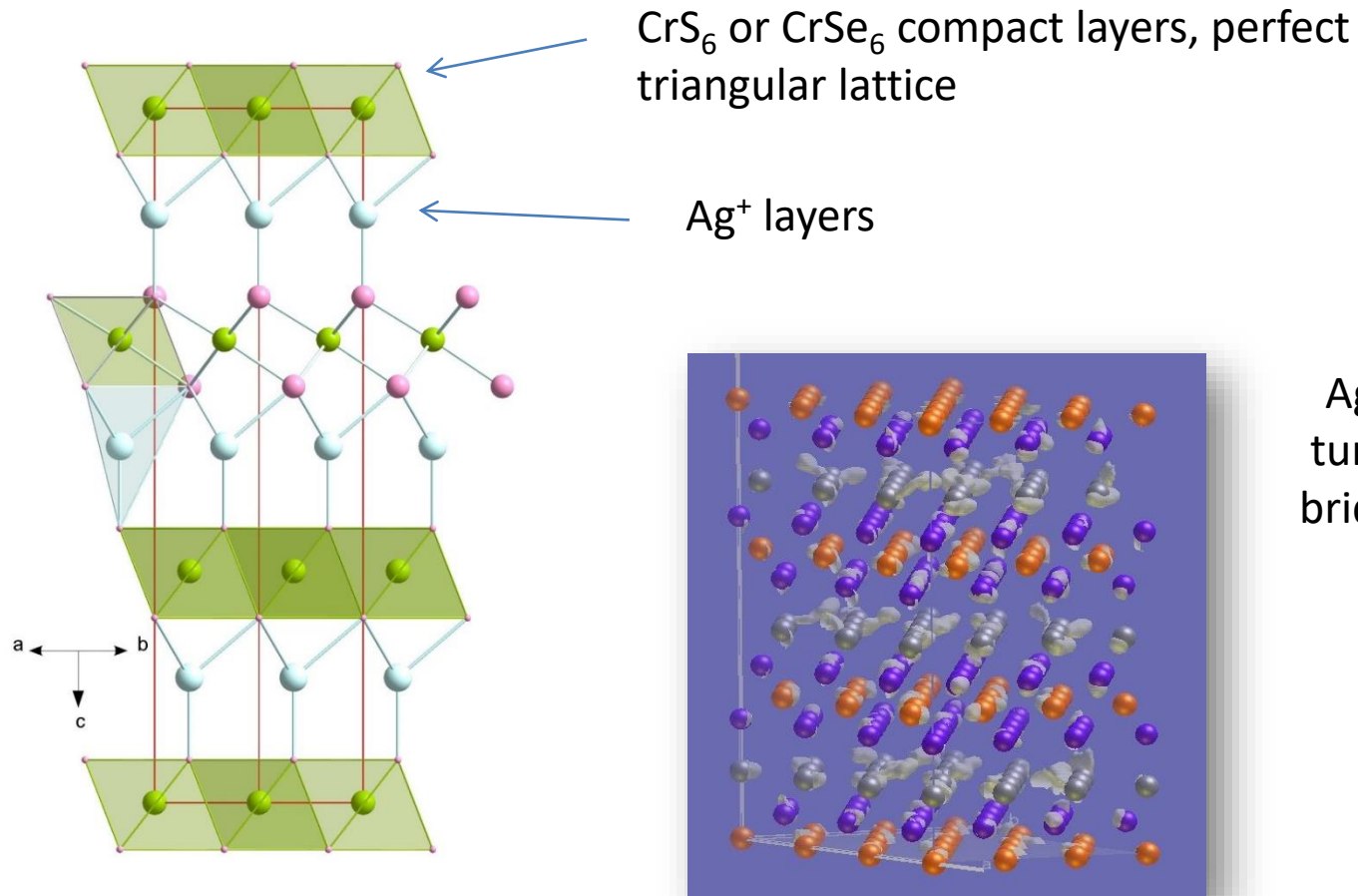


- Changing E_i to modify energy range and energy resolution



Example 3 : AgCrSe_2

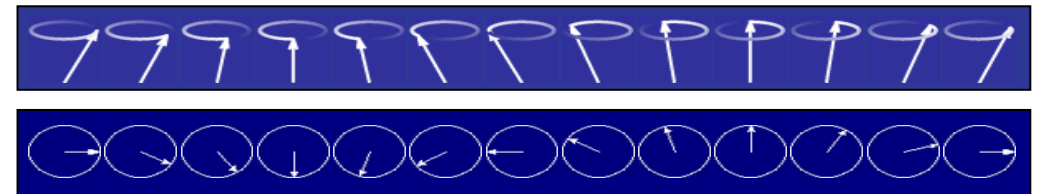
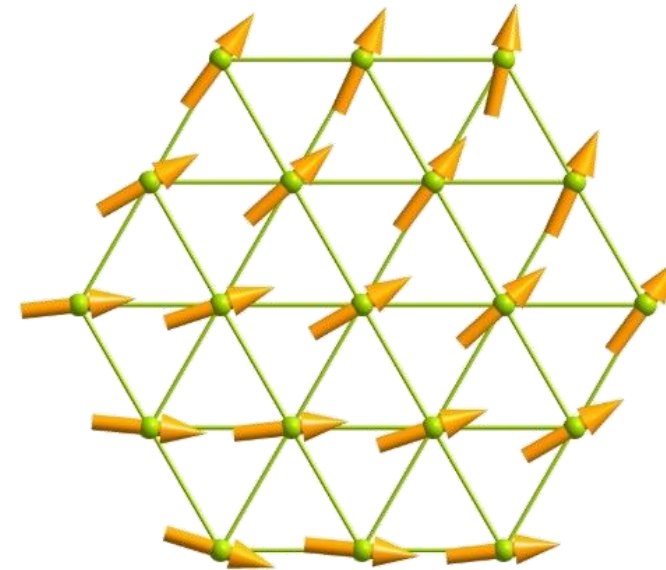
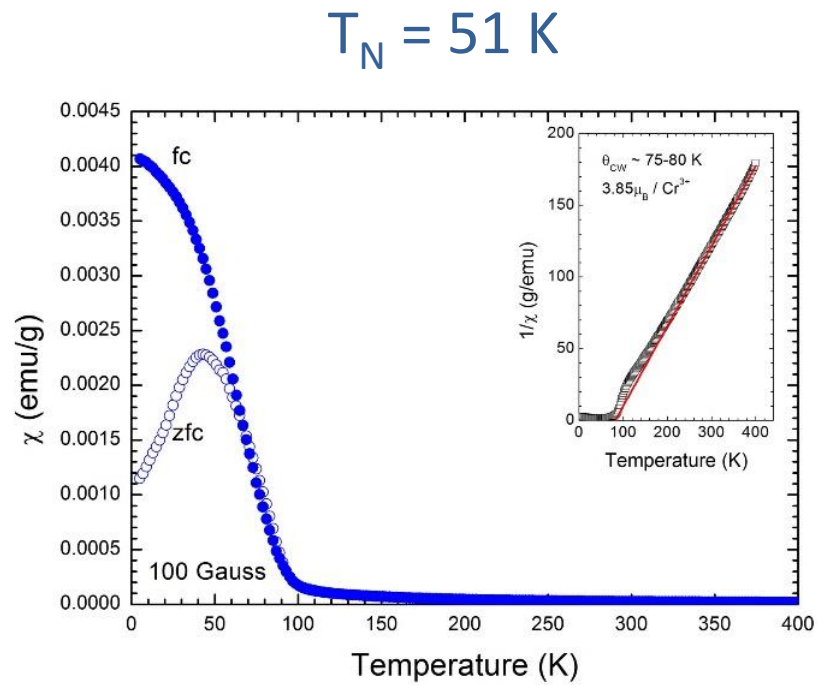
Why this example ? because the lattice dynamics is remarkable



Ag atoms (grey) have tunnel-like trajectories bridging two sites, even as low as 150 K.

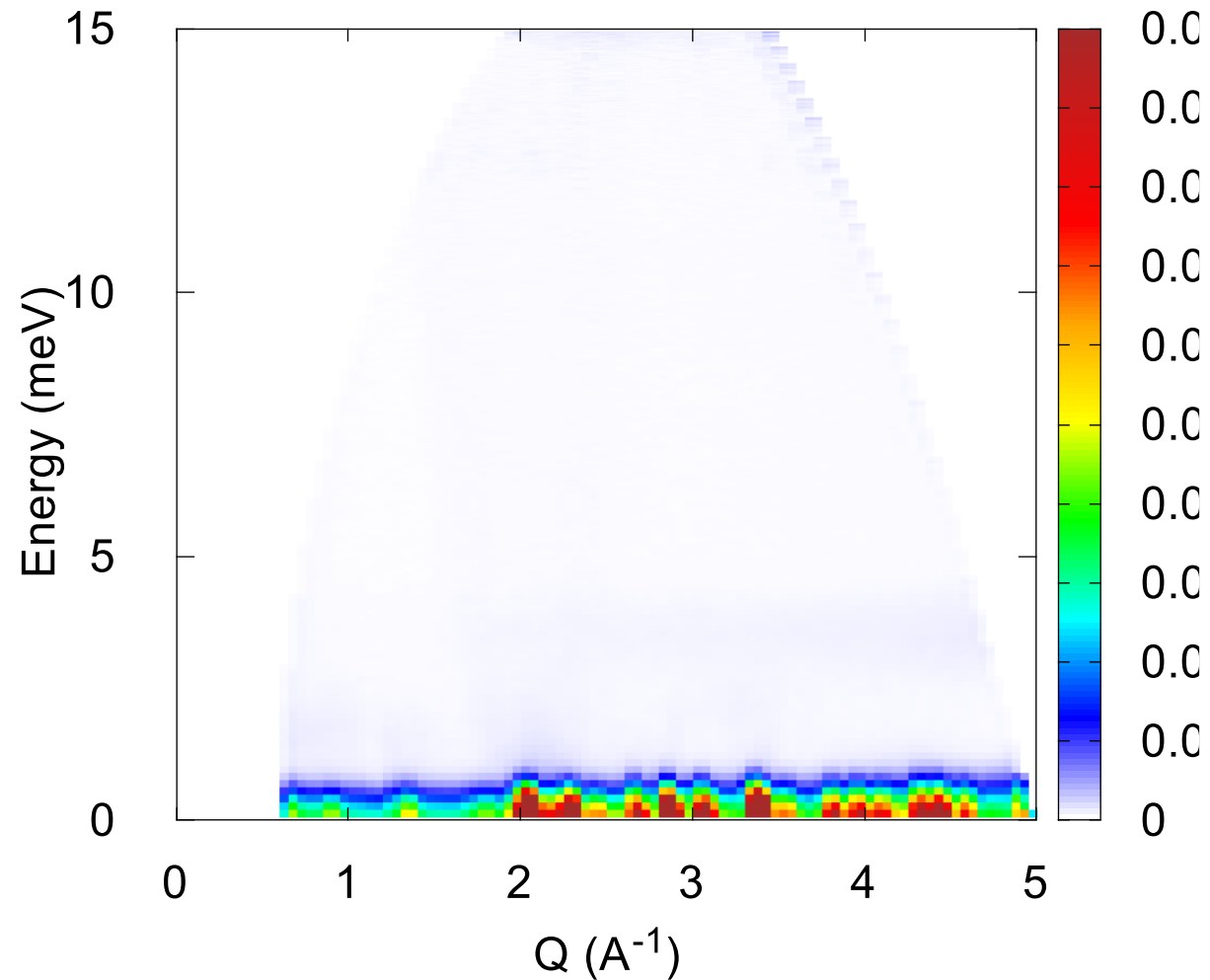
Example 3 : AgCrSe_2

And also because AgCrSe_2 is magnetic and thus hosts magnetic excitations



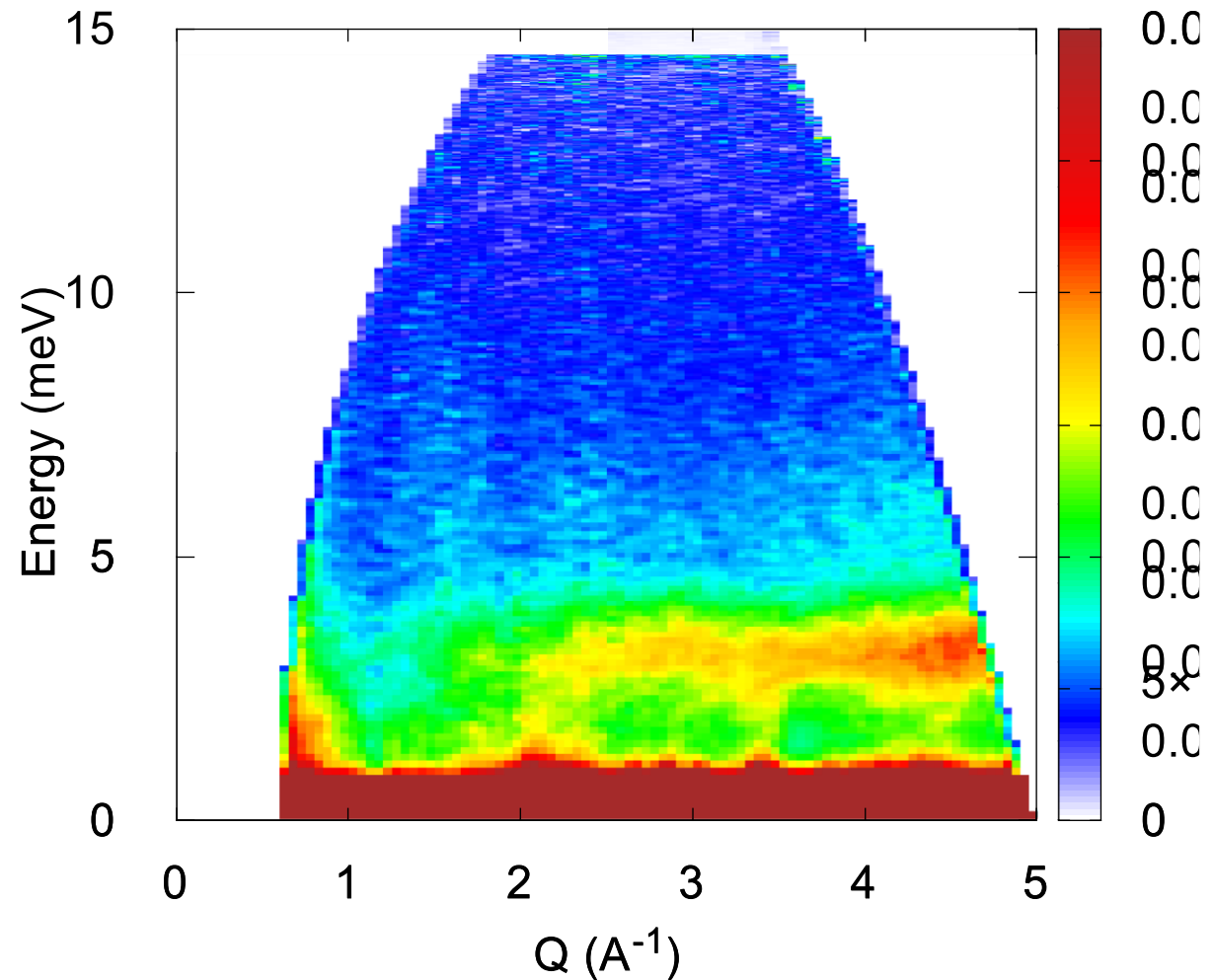
Example 3 : AgCrSe_2

Again, not much to see, except the elastic line : change color-scale !!

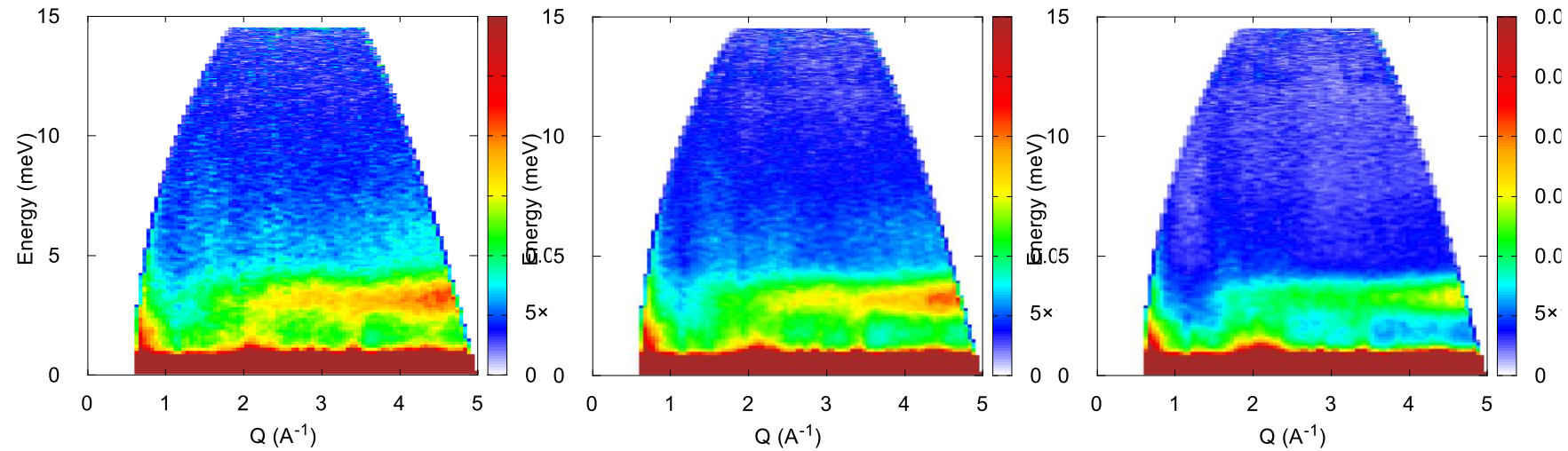


Example 3 : AgCrSe_2

Again, not much to see, except the elastic line : change color-scale !!



Example 3 : AgCrSe_2

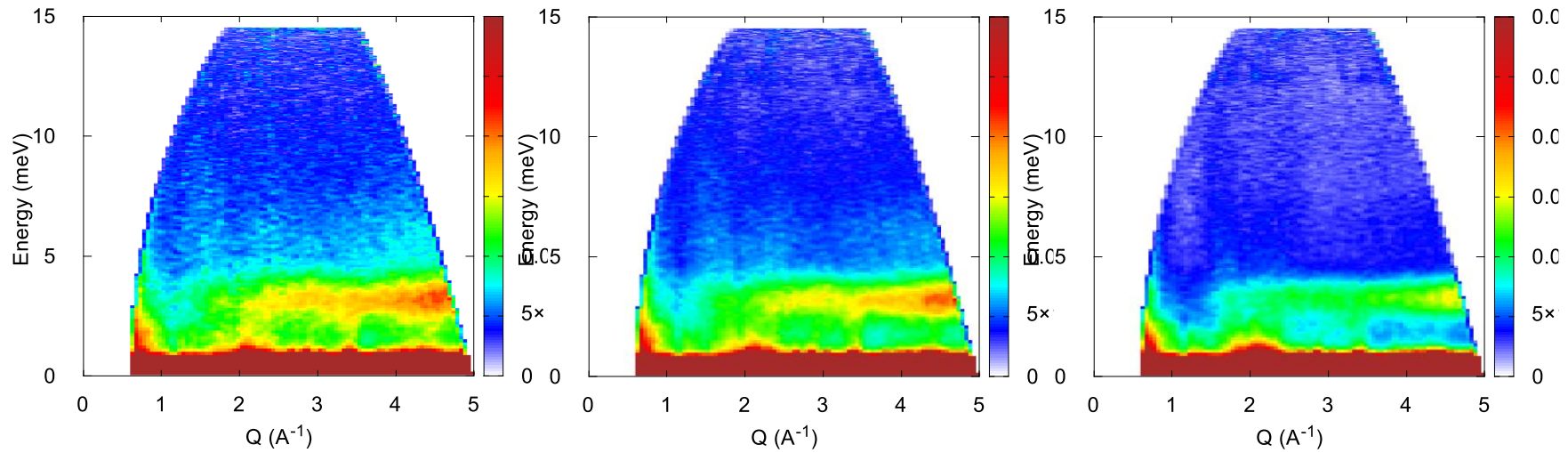


Intrinsic temperature dependence (phonons are bosons)

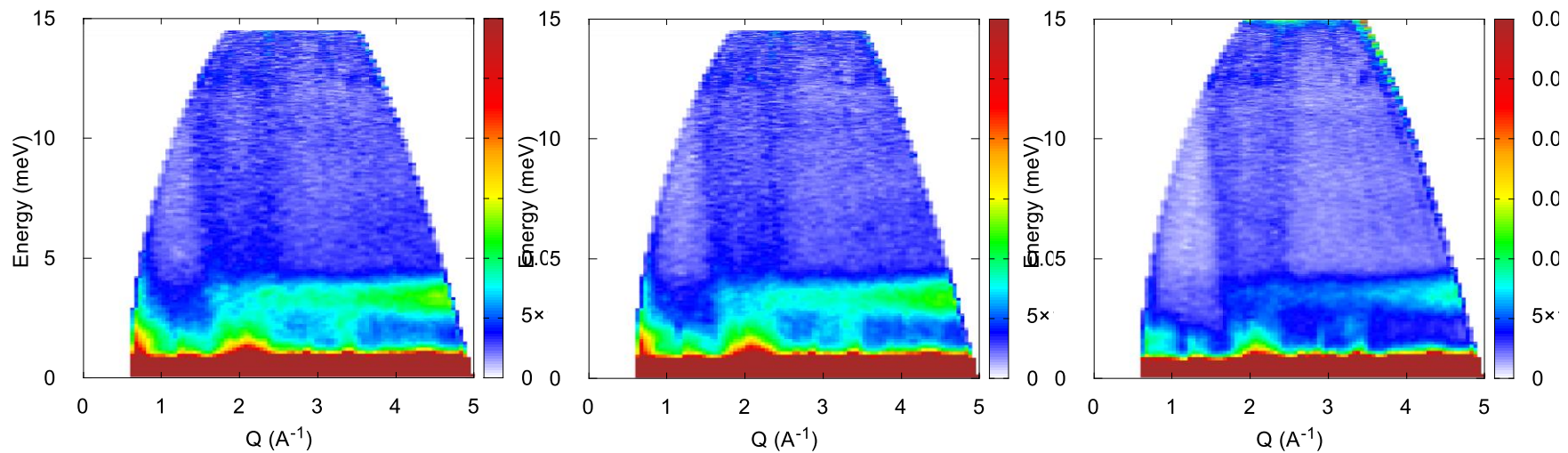
$$1 + \frac{1}{\exp E/k_B T - 1}$$

Anomalous low energy band at about 3 meV due to Ag jumps

Example 3 : AgCrSe_2

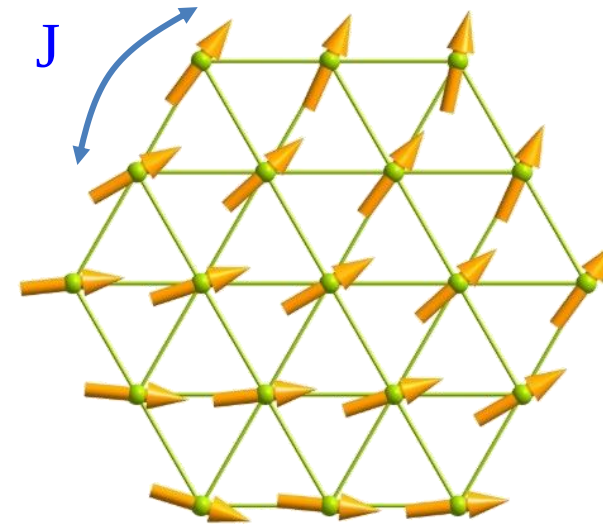
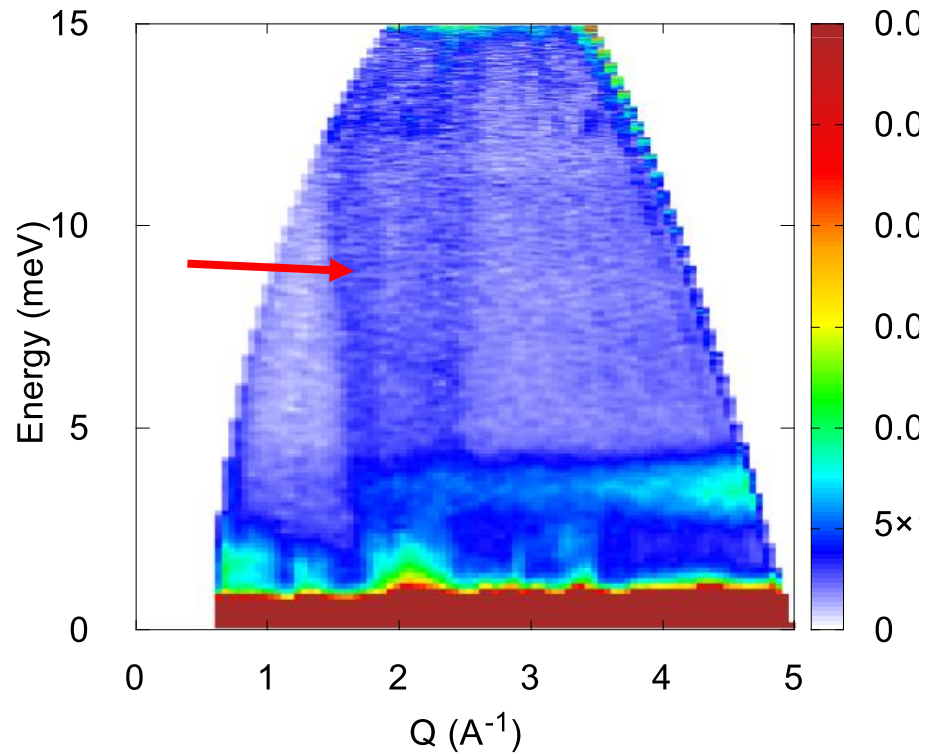


Additional signal due to Cr spins below T_N



Example 3 : AgCrSe₂

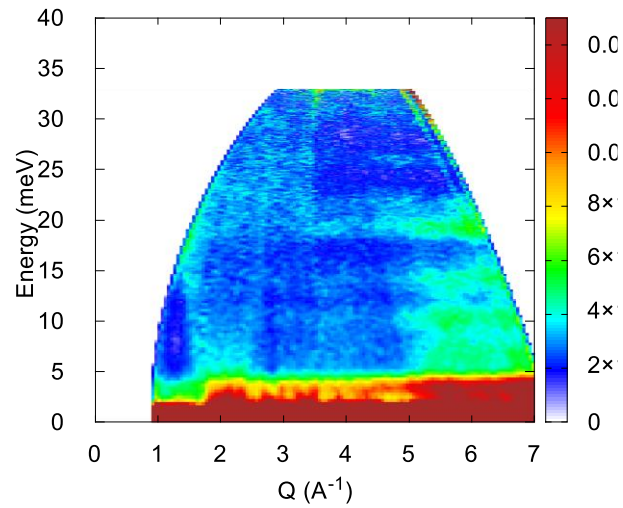
As for phonons, this is the partial density of states of magnetic excitations



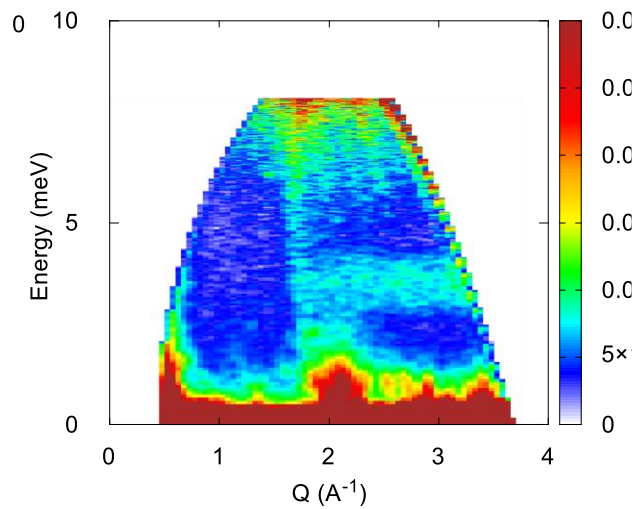
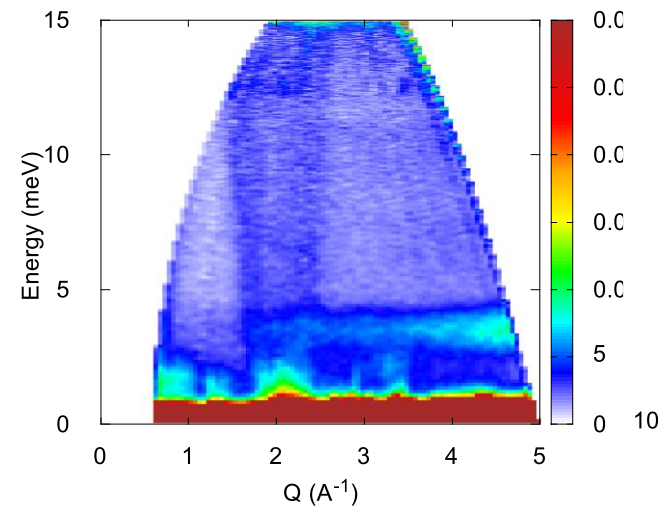
$$\mathcal{H} = \sum_{\langle ij \rangle} J(\mathbf{r}_i - \mathbf{r}_j) \mathbf{S}_i \cdot \mathbf{S}_j,$$

Determine exchange interactions between spins

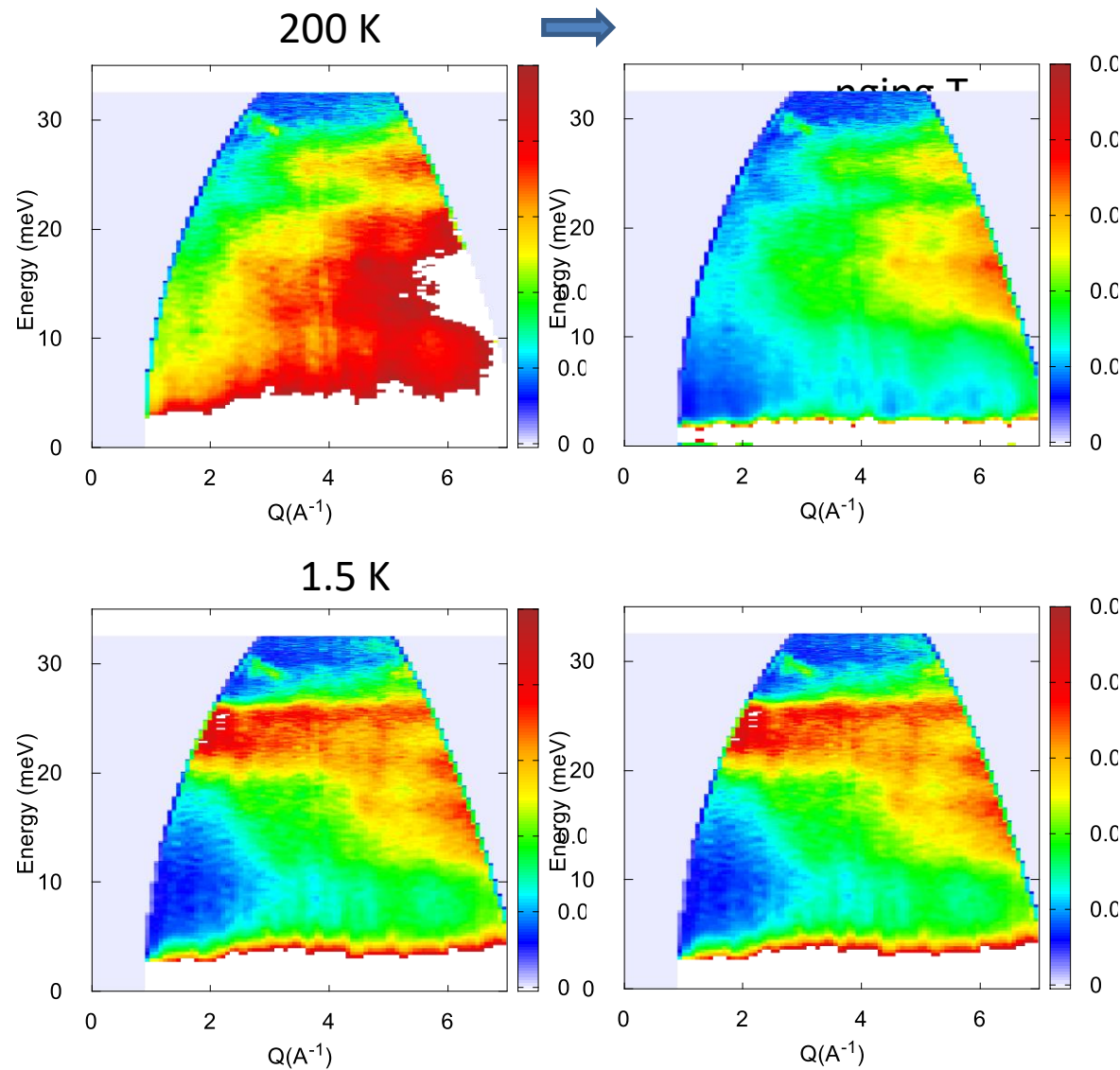
Example 3 : AgCrSe_2



Change Ei to modify energy range and energy resolution



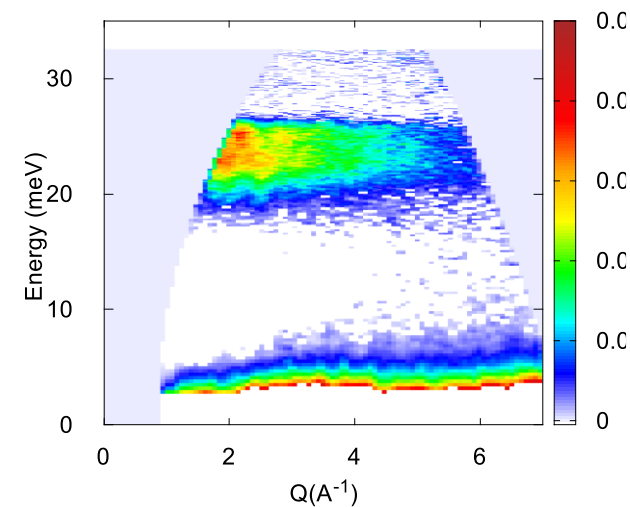
Back to YNiO₃



Correct for the Bose factor

$$1 + \frac{1}{\exp E/k_B T - 1}$$

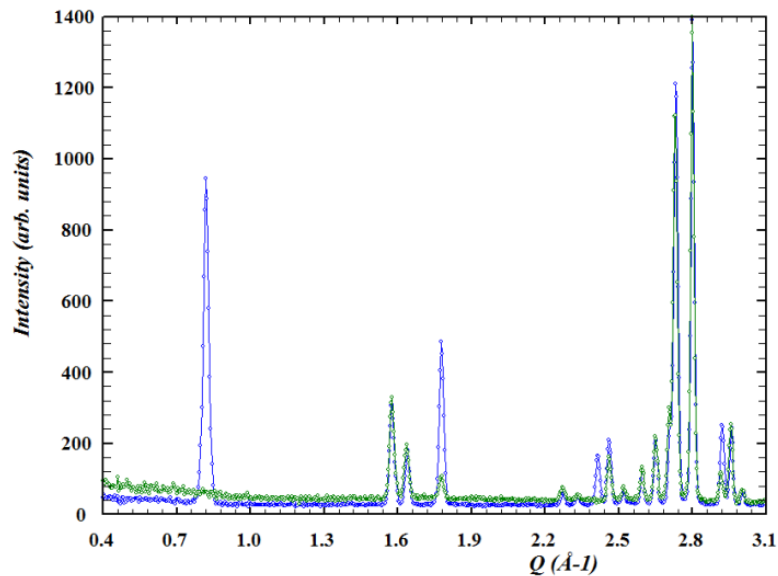
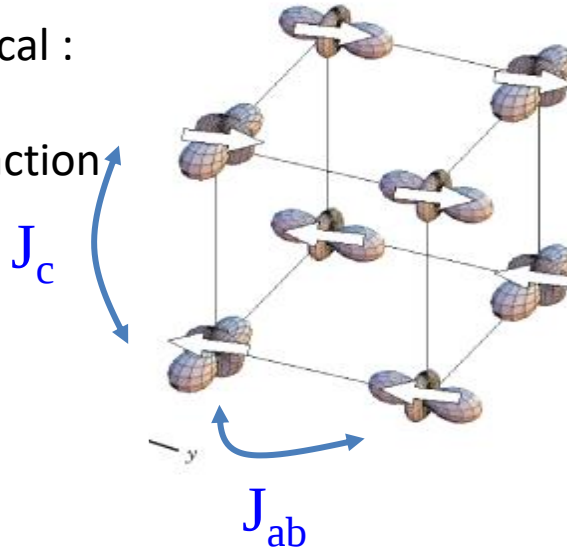
Take the difference



to reveal Ni spin excitations

Example 4 : LaMnO_3

Tomorrows' practical :
see the magnetic
transition by diffraction



But what about the
values of J_c and J_{ab} ?

$$\mathcal{H} = \sum_{\langle ij \rangle} J(\mathbf{r}_i - \mathbf{r}_j) \mathbf{S}_i \cdot \mathbf{S}_j,$$

